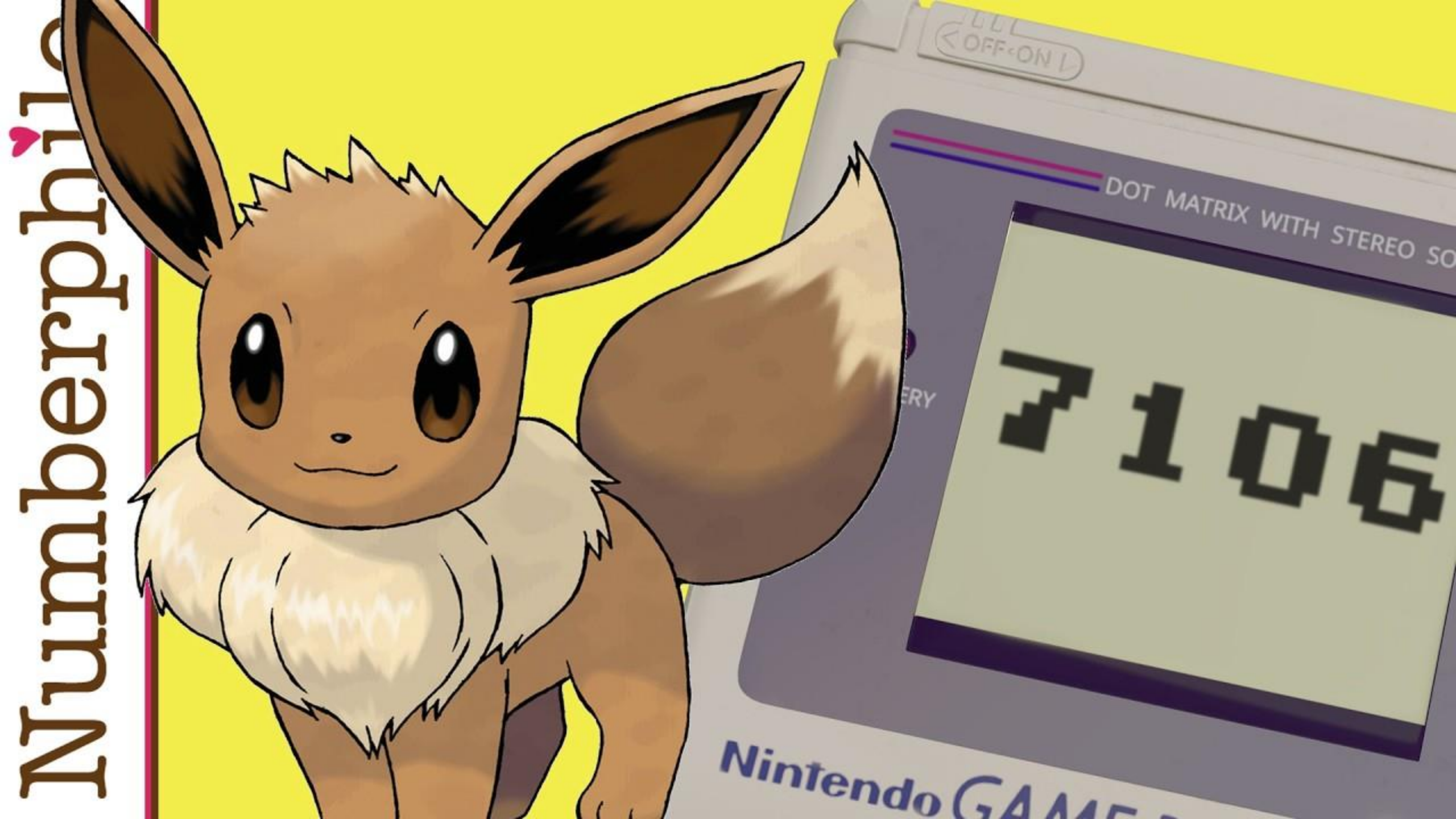
Numberphile
OFF ON
DOT MATRIX WITH STEREO SO
7106
Nintendo

$$T_1 = 1$$

$$T_2 \sim \text{Geometric}\left(\frac{n-1}{n}\right)$$

$$T_3 \sim \text{Geometric}\left(\frac{n-2}{n}\right)$$

$$\vdots$$

$$T_n \sim \text{Geometric}\left(\frac{n-(n-1)}{n}\right)$$

$$n \sum_{k=1}^{n} \frac{1}{k} = n H_n$$

Stage 0: Total E	Grass		E	Old Rod		E	Good Rod		E	Supe
Pallet Town				Magikarp	100.00%	1	Poliwag	50.00%	2	Poliwag
							Goldeen	50.00%	2	Tentacool
Route 1	Pidgey	55.00%	1.818181818							
	Rattata	45.00%	2.222222222							
Route 2	Weedle	15.00%	6.666666667							
	Pidgey	40.00%	2.5							
	Rattata	45.00%	2.222222222							
Viridian City				Magikarp	100.00%	1	Poliwag	50.00%	2	Poliwag
							Goldeen	50.00%	2	Tentacool
Route 22	Weedle	15.00%	6.666666667							
	Pidgey	45.00%	2.222222222							
	Rattata	40.00%	2.5							
Viridian Forest	Caterpie	5.00%	20							
	Metapod	5.00%	20							
	Weedle	50.00%	2							
	Kakuna	35.00%	2.857142857							
	Pikachu	5.00%	20							
Route 3	Pidgey	50.00%	2							
	Spearow	40.00%	2.5							
	Jigglypuff	10.00%	10							
Mt.Moon (1F)	Clefairy	1.00%	100							
	Zubat	79.00%	1.265822785							
	Paras	5.00%	20							
	Geodude	15.00%	6.666666667							
Mt.Moon (B1F)	Clefairy	4.00%	25							
	Zubat	60.00%	1.666666667							
	Paras	10.00%	10							
	Geodude	26.00%	3.846153846							
Mt.Moon (B2F)	Clefairy	6.00%	16.66666667							
	Zubat	54.00%	1.851851852							
	Paras	15.00%	6.666666667							
	Geodude	25.00%	4							
Route 4	Rattata	45.00%	2.222222222							
	Spearow	30.00%	3.333333333							
	Ekans	25.00%	4							

Total Unobtainable	**27/151**
Total Obtainable	**124/151**

Notes:

Legendaries count as 1 encounter each

Gift/Trade/Fossil pokemon will counter as 0 encounters since no battle occurs

Stage 0 – Geometric Distributions

- For each Pokémon in each region, model as a single Geometric distribution with probability p, expectation 1/p.
- Sum all expectations.

 # encounters = 4666

Stage 1 - Optimise

- For each Pokémon, only consider the highest encounter rate and add the expectation for that region only to the total.
- Lower bound: sum of number of expected encounters in that region for Pokémon we want
- Upper bound: sum of number of expected encounters in that region for all Pokémon

 # encounters = [850, 3615]

Stage 2 – Level up

- As for stage 1 but remove evolution lines: ie. Pokémon that ca be obtained by levelling up other species caught elsewhere

  # encounters = [340, 2336]

Stage 3 – Conditional Expectation

E.g. Consider $X \in \{1,2,3\}$ where $p_{\{1\}} = 0.5, p_{\{2\}} = 0.4, p_{\{3\}} = 0.1$

This distribution simulates Route 3, where:

P(Pidgey) = 50%, P(Spearow) = 40%, P(Jigglypuff) = 10%

To be optimal in the number of encounters required to see all Pokémon across the game, we want to observe only Spearow and Jigglypuff in this route. Therefore, we aim to calculate $E_{\{2,3\}}$.

The breakdown of this calculation is as follows:

- First, we calculate the number of expected encounters to observe any of 2 or 3 (a 'new' encounter): $\frac{1}{p_{\{1,2\}}}$
- Next, we condition on the encounter observed, and calculate the expected number of encounters to observe the other desired outcome: $p_{\{1|1,2\}}E_{\{2\}} + p_{\{2|1,2\}}E_{\{1\}}$
    - Note here that $p_{\{1|1,2\}} = \frac{p_{\{1\}}}{p_{\{1,2\}}}$, and $E_{\{2\}} = \frac{1}{p_{\{2\}}}$

Therefore, the expected number of encounters is

$$E_{\{2,3\}} = \frac{1}{p_{\{1,2\}}} + \frac{p_{\{1\}}}{p_{\{1,2\}}}\left(\frac{1}{p_{\{2\}}}\right) + \frac{p_{\{2\}}}{p_{\{1,2\}}}\left(\frac{1}{p_{\{1\}}}\right)$$

$$E_{\{2,3\}} = \frac{1}{p_{\{1,2\}}}\left(1 + \frac{p_{\{1\}}}{p_{\{2\}}} + \frac{p_{\{2\}}}{p_{\{1\}}}\right)$$

$$E_{\{2,3\}} = 10.5$$

 # encounters = [290, 302]

Total Unobtainable	**27/151**
Total Obtainable	**124/151**

Notes:

Legendaries count as 1 encounter each

Gift/Trade/Fossil pokemon will counter as 0 encounters since no battle occurs

Stage 0: Total Expected number of encounters across all pokemon encounters in all routes

Stage 1: Look at the highest encounter rate only (seeking to optimize)

Lower Bound: Sum(no. of expected encounters in that route to meet the pokemon we want)

Upper Bound: Sum(no. of expected encounters in that route to meet all pokemon in that route)

Stage 2: Remove evolutions lines (removed lines are highlighted red) - a Pokemon trainer will evolve their Pokemon! (Do not count trai

Lower Bound: Sum(no. of expected interactions in that route to meet the pokemon we want)

Upper Bound: Sum(no. of expected interactions in that route to meet all pokemon in that route)

Stage 3: Use conditional expectation - the formula will only consider encounter a 'desirable' pokemon (highlighted yellow) within each

Lower Bound: Sum(no. of expected interactions in that route to meet the pokemon we want)

Upper Bound: Sum(no. of expected interactions in that route to meet all pokemon in that route)

Results Summary		
Stage 0:		4665.653
Stage 1:	Lower Bound	849.78
	Upper Bound	3614.27
Stage 2:	Lower Bound	339.92
	Upper Bound	2335.84
Stage 3:	**Lower Bound**	**290**
	Upper Bound	**302**