

An unsolved trivial question

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Irrational beverages

- Is $e\pi$ or $e + \pi$ irrational?
- Both questions are unsolved!
- Would you like tea or coffee?
- Normal answers include: “tea”, “coffee”, “neither”. Maybe “both”?
- But we’re not normal! How about “yes”?

A logician's interpretation

- Is it true that (“you would like tea” or “you would like coffee”)?
- Is it true that (“ $e\pi$ is irrational” or “ $e + \pi$ is irrational”)?

Transcendental

- Transcendental means “not algebraic”
- Algebraic: the solution to a polynomial equation with rational coefficients eg $x^3 + 2x - 5 = 0$
- Transcendental numbers are not the solution to any such equation
- If a transcendental number is the solution, the coefficients aren't all rational



Transcendental

- Liouville (1844) proved that transcendental numbers exist
- First proven transcendental number (1851) was $\sum_{n=1}^{\infty} 10^{-n!}$
- Cantor (1874) proved that almost all reals are transcendental
- Hermite (1873) proved that e is transcendental
- Lindemann (1882) proved that π is transcendental



How does this help?

- $(x - e)(x - \pi) = x^2 - (e + \pi)x + e\pi = 0$
- e and π are both roots
- Therefore, the coefficients are not all rational

Is $e\pi$ or $e + \pi$ irrational?

Yes!