$$(x^2 - 7x + 11)^{(x^2-11x+30)} = 1.$$

$$x = 2, 5$$

$$x = 5, 6$$

$$x = 3, 4$$

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
```

In [2]:
```
def jocelyn(x):
    return pow(x*x - 7 *x + 11, x*x - 11*x + 30)
```

In [3]:
```
jocelyn(5)
```

Out[3]: 1

In [4]:
```
jocelyn(6)
```

Out[4]: 1

In [5]:
```
jocelyn(2)
```

Out[5]: 1

there is another number that is 1 when raised to some powers but not all 12:38 pm

limits? 12:38 pm

-1 12:38 pm

yep 12:38 pm

Ah I see 12:38 pm

so solutions of x*x - 7 *x + 11 = -1 12:38 pm

which would be x*x - 7 *x + 12 = 0 12:38 pm

verifying that those values yield an even numb… the other quadratic 12:39 pm

x = (7 ± √(49 - 44))/2 12:39 pm

44 should be 48 12:40 pm

are you on your phone or your computer? 12:40 pm

4 or 3 12:40 pm

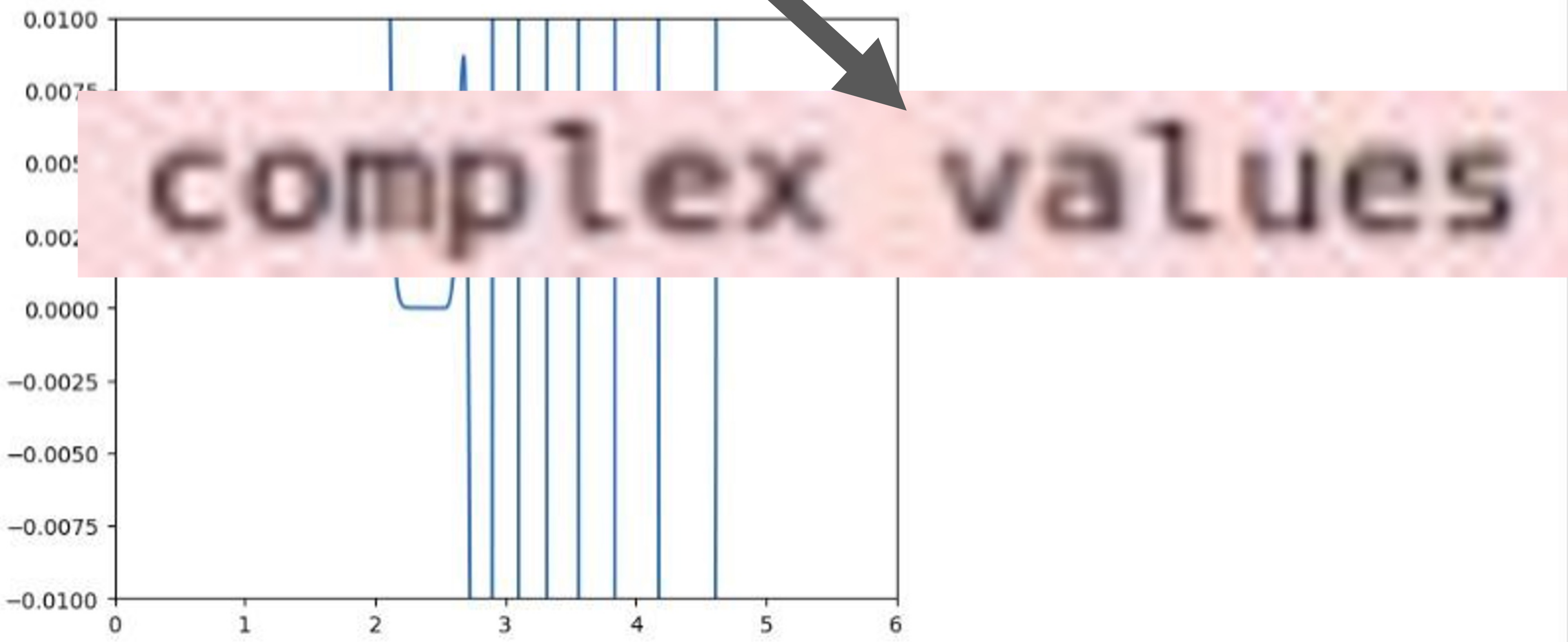
both 12:40 pm

I'm a little turned on by your fluency with the symbols 12:41 pm

```
In [6]: X = pd.Series(np.arange(0, 6, 0.001))
        Y = X.apply(jocelyn)
        plt.plot(X, Y)
        plt.xlim((0,6))
        plt.ylim((-0.01, 0.01))
```

```
/media/thomas/642d0db5-2c98-4156-b591-1a3572c5868c/anaconda3/envs/py310/lib/python3.10/site-packages/matplotlib/cbo
ok/__init__.py:1369: ComplexWarning: Casting complex values to real discards the imaginary part
  return np.asarray(x, float)
```

```
Out[6]: (-0.01, 0.01)
```



$Z \in \mathbb{C}$

$Z = 1$ where Z is complex $\Rightarrow Z = e^{i\theta}$

$\theta = (1+2n)\pi, \quad n \in \mathbb{Z}$

~~$W = Ae^{i\varphi}$, $Y = B$ — W, Y both complex~~

$V^W = 1 \qquad V, W \in \mathbb{C} \quad \Rightarrow V^W = Z$

$V = Ae^{i\varphi} \qquad W = Be^{ix}$

$V^W = (Ae^{i\varphi})^{Be^{ix}} = \left(A^{Be^{ix}}\right)\left(e^{i\varphi}\right)^{Be^{ix}}$

$e^{ix} = \cos x + i \sin x$

$V^W = \left[(Ae^{i\varphi})^B\right]^{(\cos x + i \sin x)}$

$x = a + bi$

$(x-5)(x-6)$

$(a + bi - 5)(a + bi - 6)$

$a^2 - b^2 + 2abi - 6a - 5a - 11a - 11bi$

$2ab - 11b = 0$

$b = 0$ or $a = 11/2$, $b = b$

$\frac{121}{4} - b^2 - \frac{121}{2}$

$-b^2 - \frac{121}{4}$

$x^2 - 7x + 11 = \frac{11}{2} + bi$

$\left(\frac{11}{2} + bi\right)\left(\frac{11}{2} + bi - 5\right)\left(\frac{11}{2} + bi - 6\right) = 1$

$\left(\frac{11}{2} + bi\right)^{\left(-b^2 - \frac{121}{4}\right)} = 1$

$(\ldots)^{(x^2 - 11x + 30)}$

$(x^2 - 7 + 11) = 1$

$Ae^{i\theta \, Be^{i\varphi}} = 1$

$x = c + di$

$c^2 - d^2 + 2cdi - 7c - 7di + 11 = \frac{11}{2} + b$

$c^2 - d^2 - 7c + 11 = \frac{11}{2} \rightarrow \sqrt{c^2 - 7c + \frac{11}{2}} = d$

$2cd - 7d = b$

$x = c \mp \sqrt{c^2 - 7c + 11/2}\, i$

$z^r = 1$

$\sqrt[z]{\ } =$

pm

$(Ae^{i\theta})^{a+bi} = 1$

$A^{a+bi}\left(e^{ia\theta} \times e^{-b\theta}\right) = 1$

$\left(c - \frac{7}{2}\right)^2 - \frac{49}{4}$

$\left(c - \frac{7}{2}\right)^2 < \ldots$

$\frac{7}{2} - \frac{\sqrt{21}}{2} < c < \ldots$

$(x^2 - 7 + 11)^{(x^2 - 11x + 30)} = 1$

$(Ae^{i\theta})^{Be^{i\varphi}} = 1 \qquad A, B \in \mathbb{R}$

$A^{Be^{i\varphi}} e^{i\theta B e^{i\varphi}} = 1$

$Ae^{i\theta(a+bi)} = 1 \qquad \theta = 2n\pi \quad n \in \mathbb{Z}$

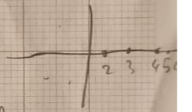
$(Ae)^{i\theta a}(Ae)^{-\theta b} = 1\, e^{i\Theta}$

$i\theta a = i\Theta = 0 \qquad a = 0$

$(Ae)^{-\theta b} = 1 \qquad Ae = 1^{\theta b} = 1$

$A = \frac{1}{e}$

$\left(\frac{1}{e} e^{i\theta}\right)^{bi} = 1$



$x^2 - 11x + 30 = bi$

$x^2 - 7 + 11 = \frac{1}{e} e^{i\theta}$

$x^2 - 11x + 30 - bi = 0$

$x = \frac{11 \pm \sqrt{121 - 4(30 - bi)}}{2}$

$x = \frac{11 \pm \sqrt{1 + 4bi}}{2}$

$x^2 - 7 + 11 - \frac{1}{e}e^{i\theta} = 0$

$x = \frac{7 \pm \sqrt{49 - 4(11 - \frac{1}{e}e^{i\theta})}}{2}$

$x = \frac{7 \pm \sqrt{5 + e^{i\theta - 1}}}{2}$

$f(x) = ax^2 + bx + c$

if $x = \mathrm{p}$
$f(x) = a\mathrm{p}^2 + b\mathrm{p} + c$

real, $\forall$ p,q

if $x = \mathrm{q}i$
$f(x) = a(\mathrm{q}i)^2 + b(\mathrm{q}i) + c$

imaginary if $c - a\mathrm{q}^2 = 0,$ otherwise complex, $\forall$ p,q

if $x = \mathrm{p} + \mathrm{q}i$
$f(x) = a(\mathrm{p} + \mathrm{q}i)^2 + b(\mathrm{p} + \mathrm{q}i) + c$
$f(x) = a(\mathrm{p}^2 + 2\mathrm{pq}i - \mathrm{q}^2) + b(\mathrm{p} + \mathrm{q}i) + c$
$f(x) = a\mathrm{p}^2 - a\mathrm{q}^2 + b\mathrm{p} + (2\mathrm{pq} + b)i$

imaginary if $c - a\mathrm{q}^2 = 0,$ real if $c - a\mathrm{q}^2 = 0$ otherwise complex, $\forall$ p,q

therefore if $f(x) = ax^2 + bx + c$, $f(\mathrm{p}+\mathrm{q}i)$ can be expressed as a complex number, so the problem can be re-written as

$(Ae^{i\theta})^{\mathrm{p}+\mathrm{q}i} = 1$
Where A, θ, p, and q are real, non-zero numbers

$(e^{\ln A}e^{i\theta})^{\mathrm{p}+\mathrm{q}i} = 1$
$(e^{\ln A}e^{i\theta})^{\mathrm{p}} \times (e^{\ln A}e^{i\theta})^{\mathrm{q}i} = 1$
$e^{\mathrm{p}\ln A} \times e^{\mathrm{p}i\theta} \times e^{\mathrm{q}i\ln A} \times e^{i\theta\mathrm{q}i} = 1$
$e^{\mathrm{p}\ln A + \mathrm{p}i\theta + \mathrm{q}i\ln A - \mathrm{q}\theta} = 1$
$\mathrm{p}\ln A + \mathrm{p}i\theta + \mathrm{q}i\ln A - \mathrm{q}\theta = 0$
Re: $\mathrm{p}\ln A - \mathrm{q}\theta = 0 \;\Rightarrow\; \mathrm{p}\ln A = \mathrm{q}\theta \;\Rightarrow\; \mathrm{p} = \frac{\mathrm{q}\theta}{\ln A}$
Im: $\mathrm{p}i\theta + \mathrm{q}i\ln A = 0 \;\Rightarrow\; \frac{\mathrm{q}\theta}{\ln A}\theta + \mathrm{q}\ln A = 0$
$\mathrm{q}(\theta)^2 + \mathrm{q}(\ln A)^2 = 0$
$\mathrm{q}(\theta^2 + (\ln A)^2) = 0$

either q=0 or $\theta^2 + (\ln A)^2 = 0$
but q$\neq 0$ and if θ and A are non-zero and real, $\theta^2 \neq -(\ln A)^2$

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