The Longest Chess Game?

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MathsJam Gathering 2021

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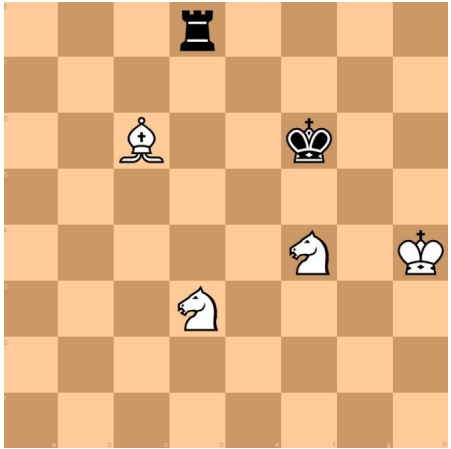
- In 1561, the “fifty-move rule” was introduced.
- (This rule only applied to games started after its introduction)
- Also, if the same position is repeated 3 times, either player can claim a draw.

50-Move Rule Example

- On move 63 of a game between Karpov and Kasparov in 1991...



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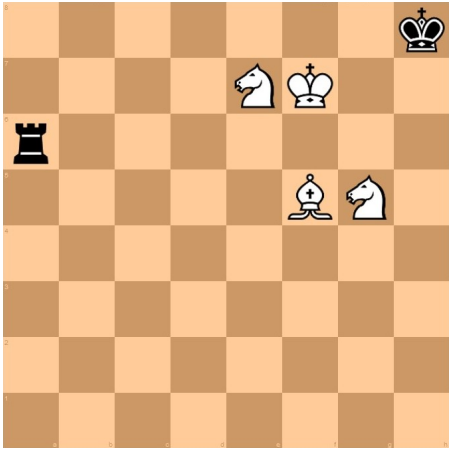
- And on move 114 of the same game...



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- In the current rules, even if no player claims it, the game is a draw:
  - if the players make 75 moves with no pieces being captured and no pawns advancing.
  - if the same position is repeated 5 times.

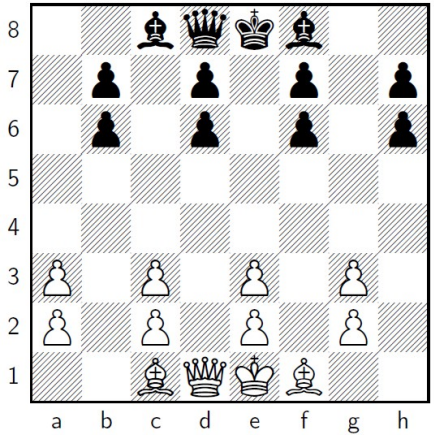
Upper Bound

- A ‘critical’ move is a pawn advance or piece capture - these moves prevent us from hitting the 75-move rule.
- There are 16 pawns, each needing 6 moves to get promoted.
- Apart from the pawns, there are 14 capturable pieces.
- And once the 16 pawns have been promoted they can be captured.

$$\underbrace{(75 + 74 + 1)}_{\substack{\text{max. possible} \\ \text{half-moves per} \\ \text{critical move}}} \times \underbrace{(16 \times 6 + 14 + 16)}_{\text{no. critical moves}} = 150 \times 126 = 18{,}900$$

Upper Bound - Pawn Advances

- In the upper bound, 76% of the critical moves were pawn advances.
- We don't want the pawns to block each other.



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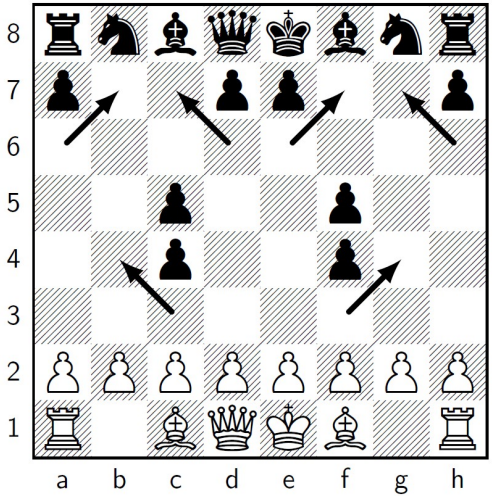
- The upper bound is now $18{,}900 - 1{,}200 = 17{,}700$ half-moves.

Upper Bound - Parity

- If we keep using the same colour for critical moves $\rightarrow$ maximally inefficient 👍.
- Every time we change the parity, we are guaranteed to lose a half-move 👎.
- We have to change parity at least once, so the upper bound is

$$17{,}700 - 1 = 17{,}699$$

Tom Murphy VII's Solution



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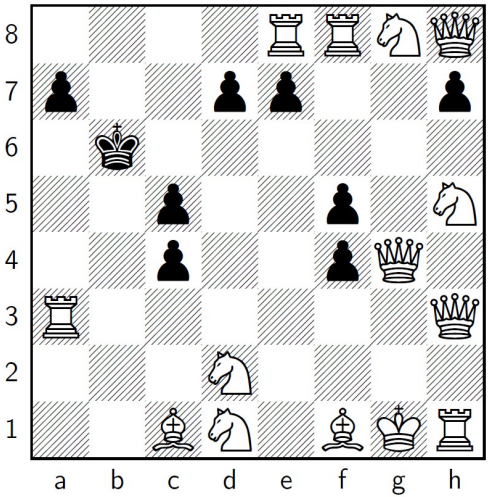

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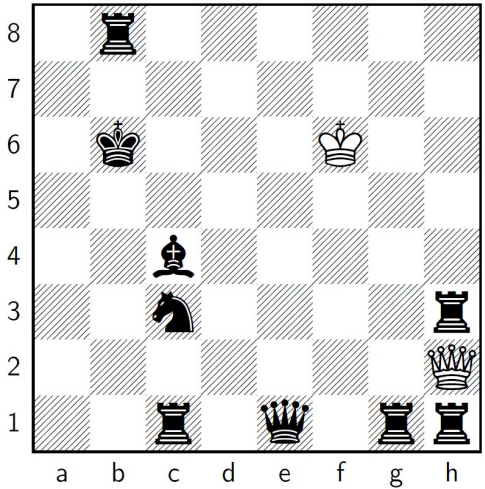

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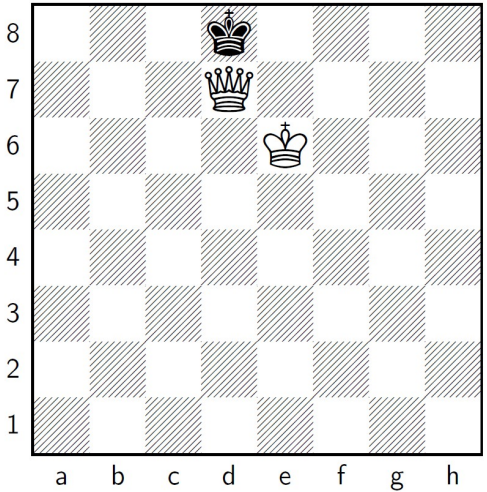

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- Can it be done with fewer parity changes?

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- It's been running for a year, so it only has 7 years left to run.