Missing Pythagoras

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All paintings by Eugen Jost

https://everything-is-number.net/

Pell equation

$2x^2 = y^2 \pm 1$

Pell($\pm$)

Pell($-$)

Pell($+$)

Pell(±)

(1,1)
(2,3)
(5,7)
(12,17)

Recursion
$x_n = 2x_{n-1} + x_{n-2}$

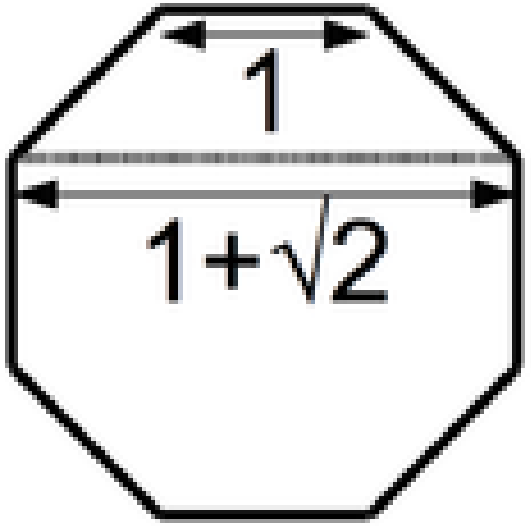
Fibonacci for grown-ups

Silver Ratio

Golden ratio φ:

$\lim \frac{F_n}{F_{n-1}} = \varphi =$ Golden ratio

$\lim \frac{x_n}{x_{n-1}} = \psi = 1 + \sqrt{2} =$ Silver ratio



Pell(−)

$2x^2 = y^2 - 1$

y odd, replace with
$y = 2y' + 1$ etc

Solutions correspond exactly to the triangle numbers that are square.

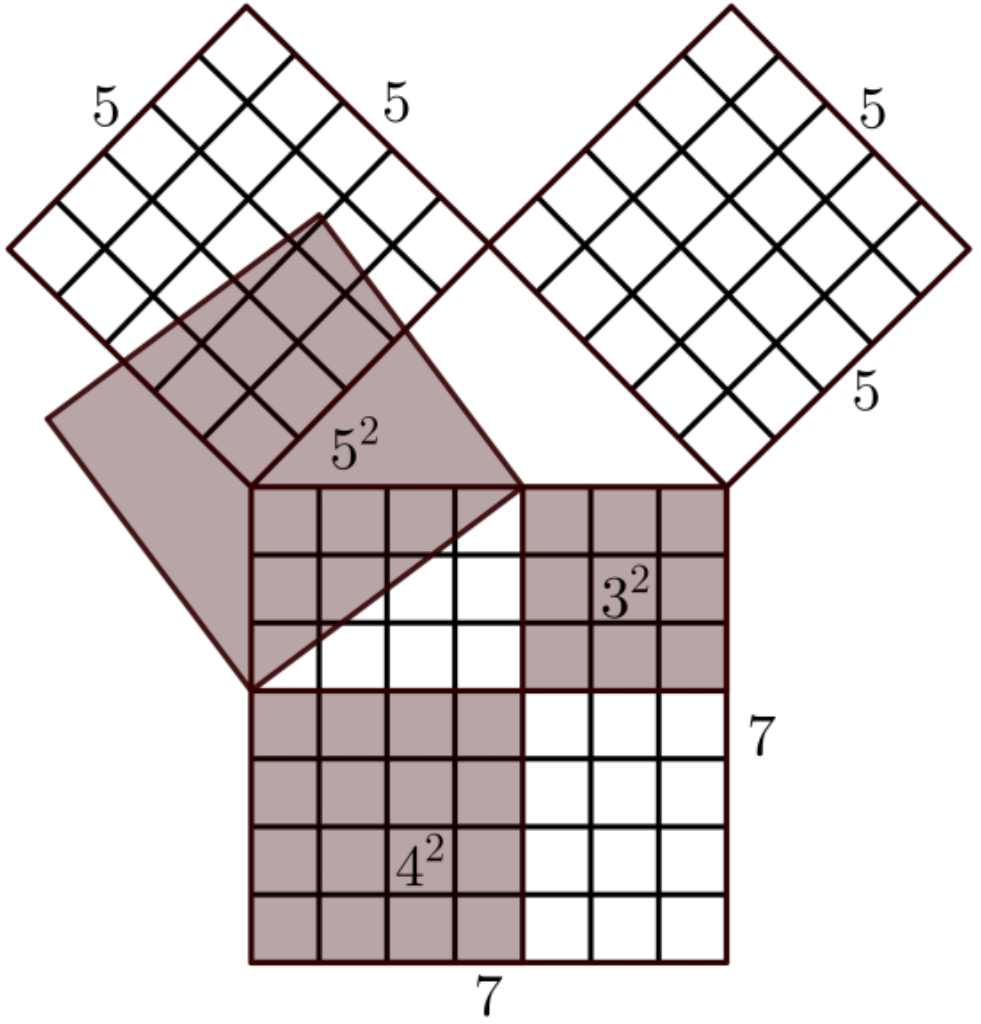
Pell(+)

$2x^2 = y^2 + 1$

Same trick

Solutions correspond exactly to Pythagorean triples where the legs differ by 1.

Pythagoras ⇔ fake Pythagoras



Question

Anyone knows
anything about

$x^2 + y^2 = z^2 \pm 1$?

Eugen Jost:
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