

nimsticks: making games with LaTeX and Lua

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Nim

- ▶ A game for two players.
- ▶ Players take turns to remove objects from several piles, subject to some rules.
- ▶ The last player to remove an object is the winner (or, sometimes, the loser).

How to draw it in \LaTeX ?

- ▶ Game Theory and Recreational Mathematics.

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How to draw it in \LaTeX ?

- ▶ Game Theory and Recreational Mathematics.

- ▶ 

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- ▶ I made my own: 

- ▶ Drawn with a little random wobble so they look ‘thrown together’ and not too regular.

nimsticks \LaTeX package

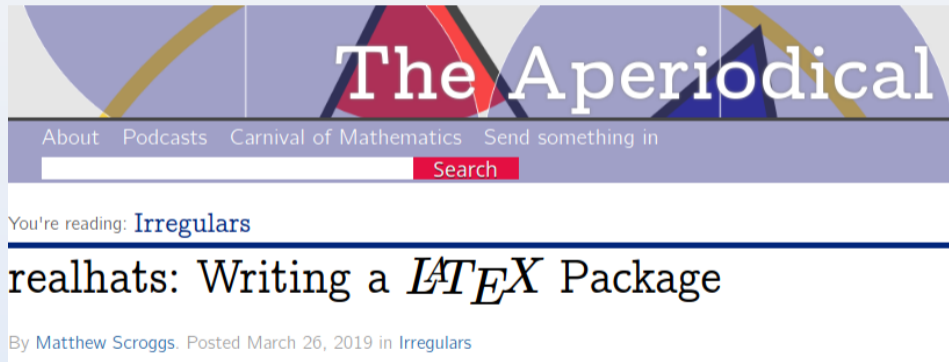
- ▶ `\usepackage{nimsticks}`
- ▶ `\nimgame{}`: takes a comma-separated list of numbers and draws a Nim game, like `\nimgame{3,4,7,1,4}` draws this:

||| |||| ||||| | //||

- ▶ This is available in TeXLive 2020 and MiKTeX should install it when you first use it.

Did you say a \LaTeX package?

- ▶ A good resource on how to make a \LaTeX package is 'realhats: Writing a \LaTeX Package' by Matthew Scroggs (search at aperiodical.com).



Lua \LaTeX

- ▶ You do use Lua \LaTeX , don't you?
- ▶ This replaces pdf \LaTeX with something more modern, but also has the Lua programming language built in.

```
\documentclass{article}
\usepackage{luacode}
\begin{document}
\begin{luacode*}
    for i = 1, 10 do
        if (i % 2 ~= 0) then
            tex.print("Hmm, " .. i .. " is a bit odd\\\\"")
        end
    end
\end{luacode*}
\end{document}
```

Nim-sum

- ▶ The nim-sum of k non-negative integers x_1, x_2, \dots, x_k , written $x_1 \oplus x_2 \oplus \dots \oplus x_k$, is obtained by adding the numbers in binary without carrying (an operation called exclusive-or or XOR).

| | | |
|----------|----------|----------|
| \oplus | 0 | 1 |
| 0 | 0 | 1 |
| 1 | 1 | 0 |

How to win at Multi-pile Nim

- ▶ If the nim-sum of the heap sizes is zero at the end of your turn, you can force a win.
- ▶ If the nim-sum of the heap sizes is not zero at the start of your turn, you can make it zero and force a win.

Example

///\ \\ //\\

Example

$$\begin{array}{r} 1 \ 0 \ 1 \\ 0 \ 1 \ 1 \\ \oplus 1 \ 0 \ 0 \\ \hline 0 \ 1 \ 0 \end{array}$$

Example

So $101 \oplus 11 \oplus 100 = 10$.

Example

So $101 \oplus 11 \oplus 100 = 10$.

Note that we have

commutativity: $a \oplus b = b \oplus a$;

associativity: $(a \oplus b) \oplus c = a \oplus (b \oplus c)$;

Example

$$\text{So } 101 \oplus 11 \oplus 100 = 10.$$

So here

$$101 \oplus 11 \oplus 100 \oplus 10 = 10 \oplus 10$$

$$101 \oplus 11 \oplus 100 \oplus 10 = 0$$

$$101 \oplus (11 \oplus 10) \oplus 100 = 0$$

$$101 \oplus 1 \oplus 100 = 0$$

Example

So $101 \oplus 11 \oplus 100 = 10$.

So here

$$101 \oplus 11 \oplus 100 \oplus 10 = 10 \oplus 10$$

$$101 \oplus 11 \oplus 100 \oplus 10 = 0$$

$$101 \oplus (11 \oplus 10) \oplus 100 = 0$$

$$101 \oplus 1 \oplus 100 = 0$$

So change the 3 pile to a 1.

Example

$$\begin{array}{r} I \backslash \backslash \quad / \quad \backslash \backslash \\ 1 \quad 0 \quad 1 \\ 0 \quad 0 \quad 1 \\ \oplus \quad 1 \quad 0 \quad 0 \\ \hline 0 \quad 0 \quad 0 \end{array}$$

A Lua \LaTeX program - <https://github.com/prowlett/nim-next-move>

This is what my Lua code does, to make documents like this

Questions

1. What is the optimal next move in the following game of Nim?

 $\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}$ $\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}$ $\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}$ $\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}\text{\textbackslash}$

2. What is the optimal next move in the following game of Nim?

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Answers

1. Nim sum is zero. There is no optimal move, the next player loses.
2. Nim sum is not zero. Change heap 3 from 3 to 2