

MathsJam 2020

Approximating Simple Ratios
with Irrational Numbers

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MathsJam 2020

You what?

- Let us consider a few simple ratios
- We can write them as fractions

$$3/2$$

$$5/4$$

$$8/5$$

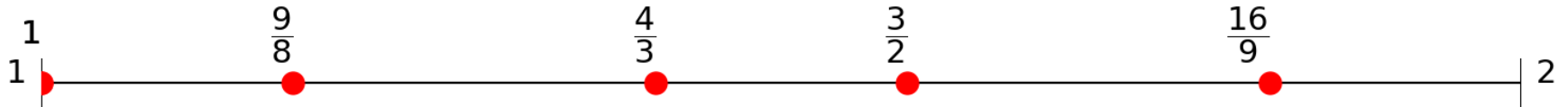
OK, now what?

- Let's normalise them so that they fall between 1 and 2
- Multiply by a power of 2

$$1/3 \rightarrow 4/3$$

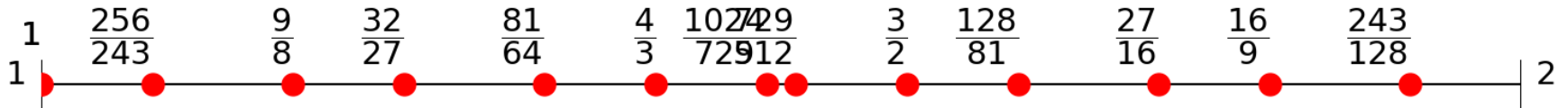
How do we make simple ratios?

- Let's try powers of 3
..., $1/3$, 1 , 3 , ...
- (normalised)
- Plot on a logarithmic number line



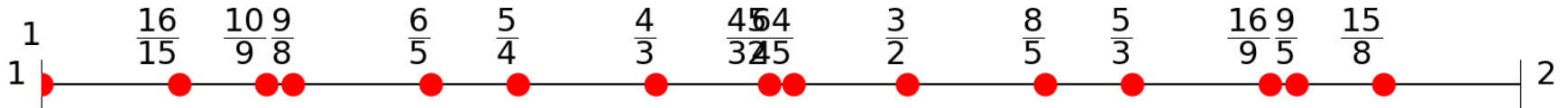
What about some more?

- More powers of 3
..., $1/3$, 1, 3, ...
- (normalised)



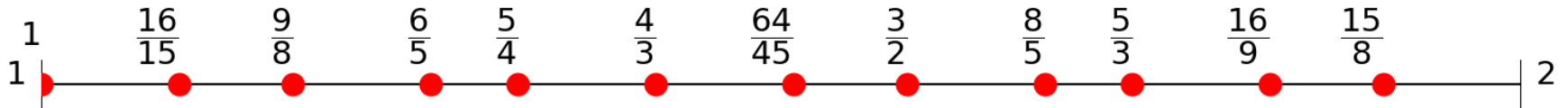
What about other bases?

- Let's try powers of 3 *and* powers of 5
..., $1/3$, 1, 3, ...
..., $1/5$, 1, 5, ...
- (multiplied, then normalised)



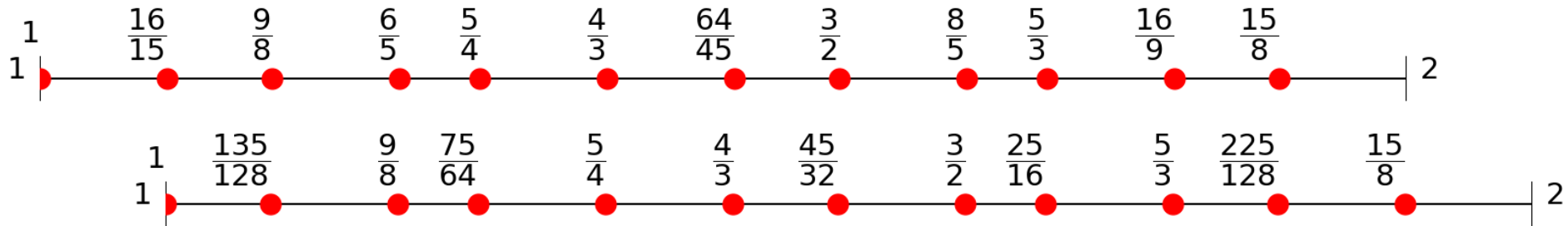
Can we make a machine?

- Let's imagine a mechanical device that can produce these ratios
- Eliminate a few options from clusters for practicality
- We have ratios spread across the whole range



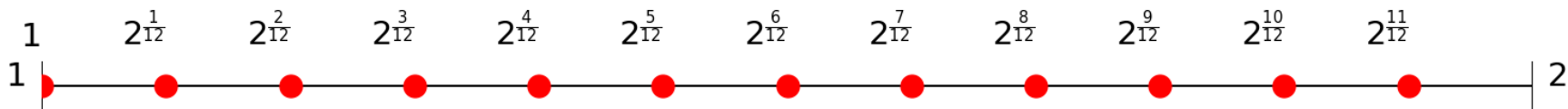
Wrong button?

- Imagine a row of buttons, and starting in the wrong place
- What ratios would you get then?



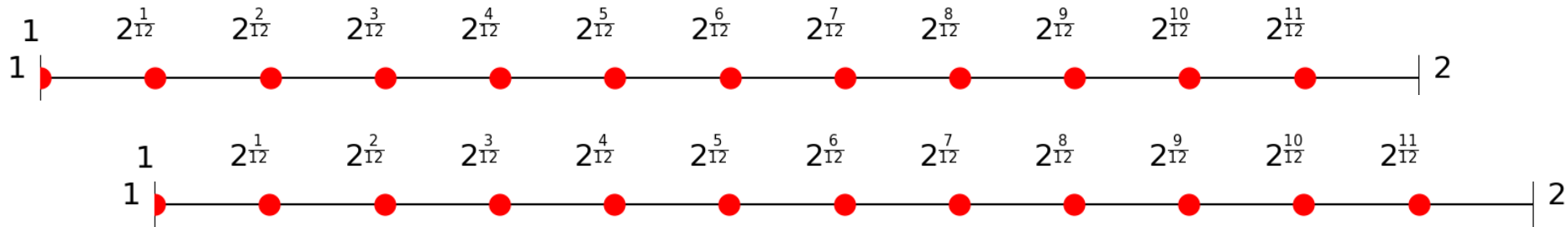
Approximate?

- Can we do something so that we get the same result no matter which button we start with?
- Let's use irrational numbers as approximations!



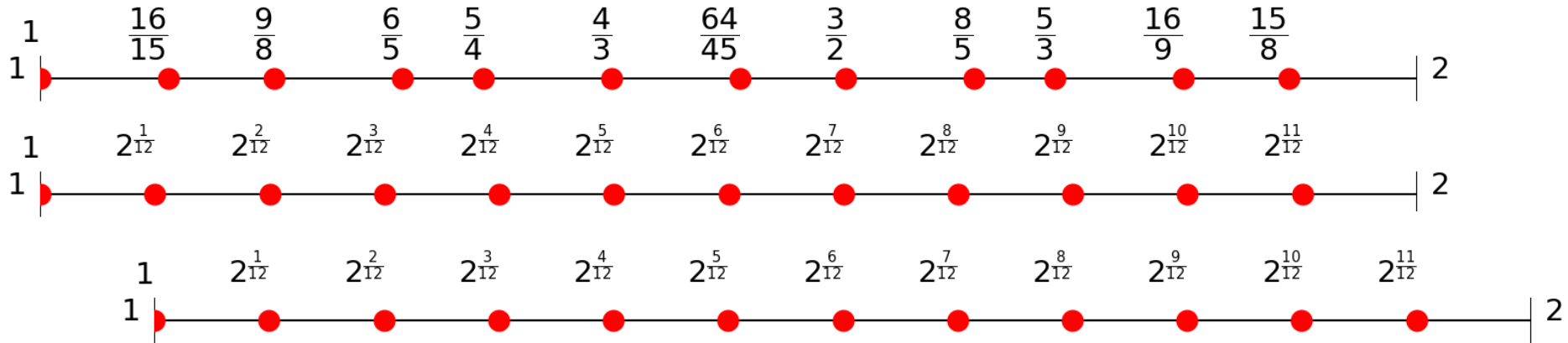
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Wow, really?

- Just Intonation
 - Powers of 3 → 3-limit Tuning / Pythagorean Tuning
 - Powers of 3 and 5 → 5-limit Tuning
- Equal Temperament
 - Roots of 2

The End

- This has been your introduction to Music Theory
(Western music from the last few centuries only)
- Thank you for listening
- Happy approximating!