Ubiquitous Permutations

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Last year at the MathsJam Gathering...

Milk (or Klondike) shuffle:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 6 & 7 & 5 & 8 & 4 & 9 & 3 & 10 & 2 & 11 & 1 & 12 \end{bmatrix}$$

Milk shuffle with drop:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 7 & 6 & 8 & 5 & 9 & 4 & 10 & 3 & 11 & 2 & 12 & 1 \end{bmatrix}$$

The notation lists the position of the objects before and after the permutation.

On the way to the Gathering...

(Actual photo of Stoke-on-Trent train station)

Φlipp Reinhard:

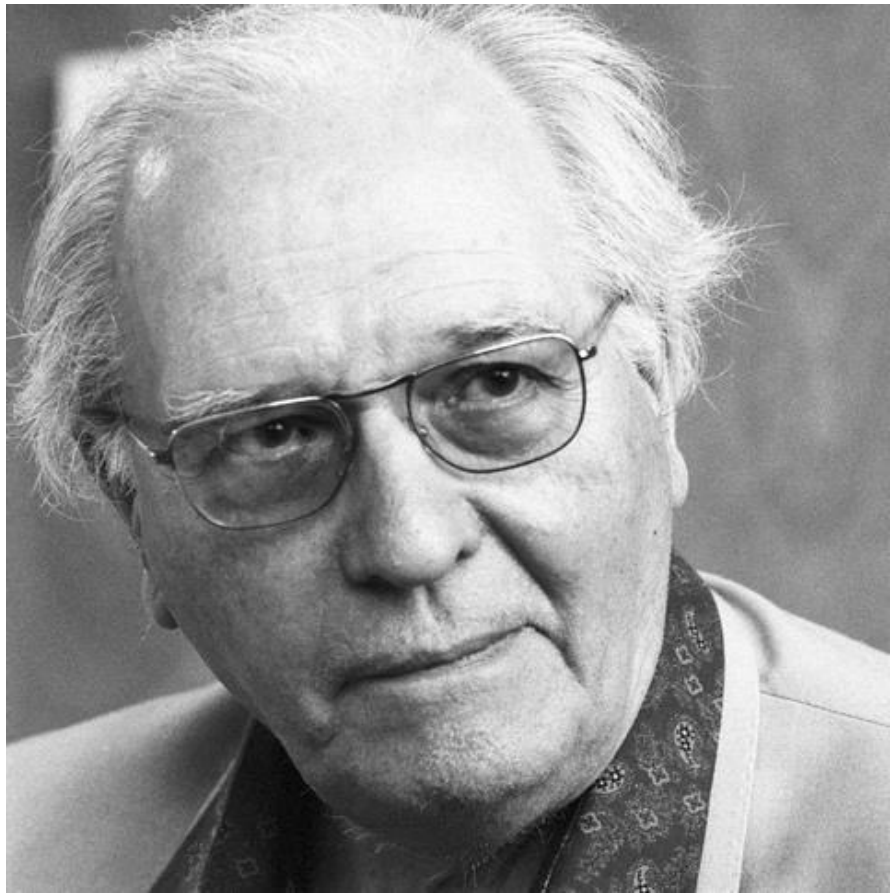
- Have you heard about this piece by Messiaen?

L’île de feu 2

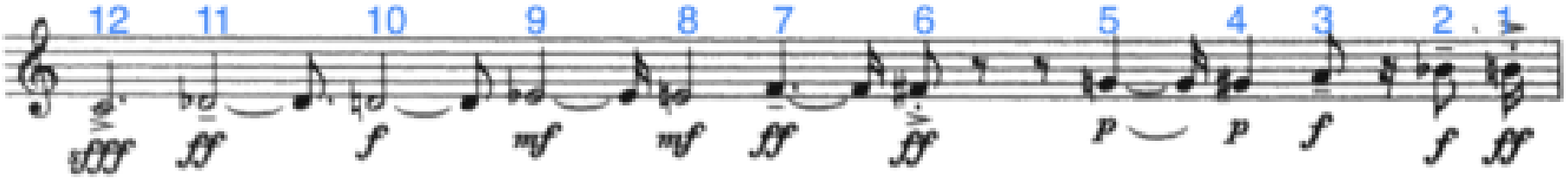
Messiaen developed a system, called **interversions** for generating the

pitches, durations and intensities

of the notes in the piece *L’île de feu 2* (*The island of fire 2*), the fourth movement of *Quatre études de rythme* (*Four Rhythm Studies*, 1949/50)**.**



Interversions



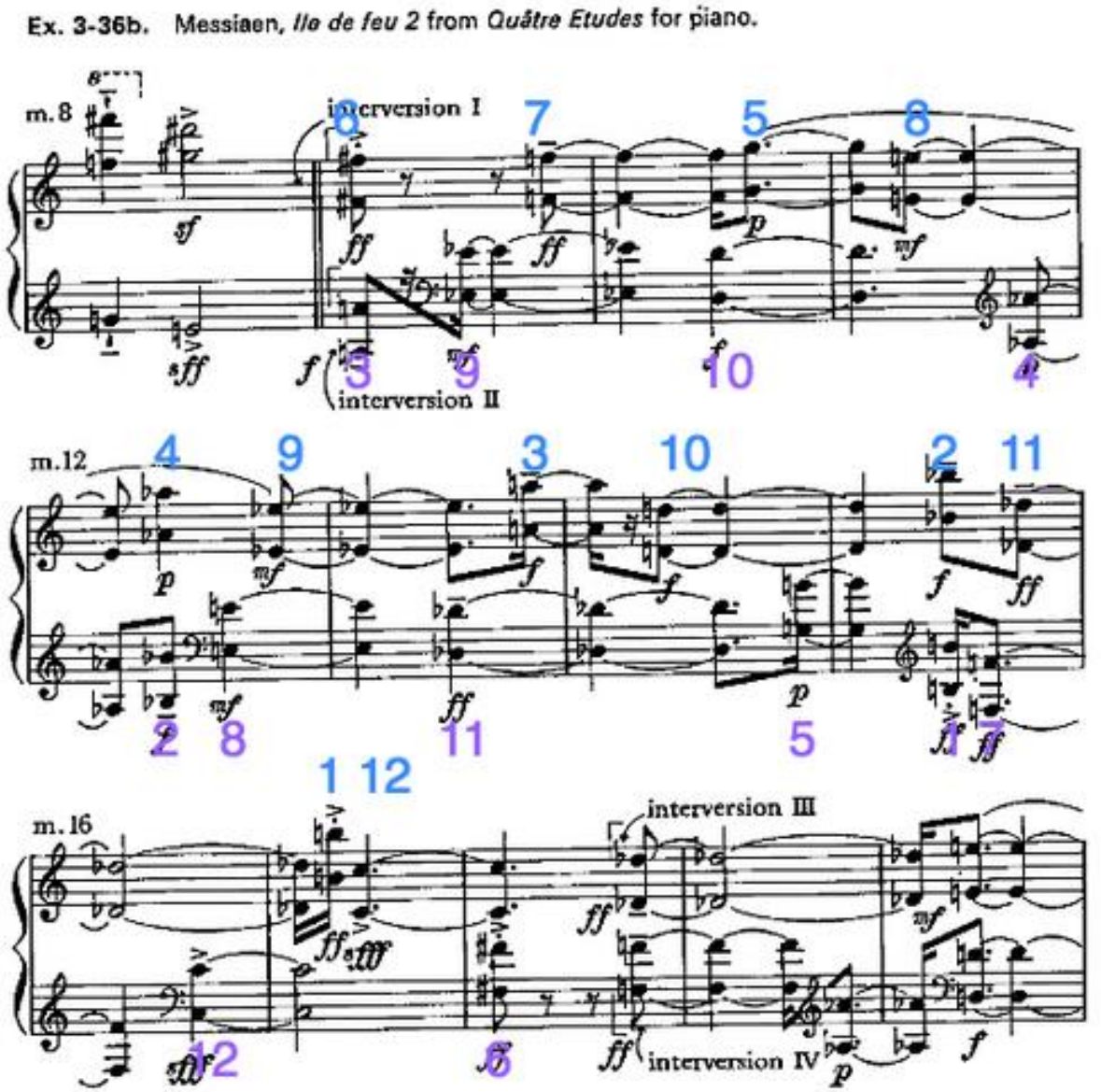
Original:	12 11 10 9 8 7 6 5 4 3 2 1
Interversion I:	6 7 5 8 4 9 3 10 2 11 1 12
Interversion II:	3 9 10 4 2 8 11 5 1 7 12 6
Interversion III:	11 8 5 2 1 4 7 10 12 9 6 3
Interversion IV:	7 4 10 1 12 2 9 5 6 8 3 11
Interversion V:	9 2 5 12 6 1 8 10 3 4 11 7
Interversion VI:	8 1 10 6 3 12 4 5 11 2 7 9
Interversion VII:	4 12 5 3 11 6 2 10 7 1 9 8
Interversion VIII:	2 6 10 11 7 3 1 5 9 12 8 4
Interversion IX:	1 3 5 7 9 11 12 10 8 6 4 2
Interversion X:	12 11 10 9 8 7 6 5 4 3 2 1

This is the milk shuffle with drop!

You can actually use it to generate the interversions.

The score

Interversion I: 6 7 5 8 4 9 3 10 2 11 1 12
Interversion II: 3 9 10 4 2 8 11 5 1 7 12 6
Interversion III: 11 8 5 2 1 4 7 10 12 9 6 3
Interversion IV: 7 4 10 1 12 2 9 5 6 8 3 11



The Mathieu Group M_{12}

Theorem. Every finite simple group is isomorphic to groups in one of 3 infinite families, or to a list of 26 sporadic groups.

The Mathieu groups M_{11} M_{12} M_{22} M_{23} and M_{24} are part of the sporadic groups.

The Mathieu group M_{12} has 95040 elements and is generated by the permutations that give the milk shuffles:

simple: (1 11 10 8 4 5 3 7 3 9 6)(12) and with drop: (1 12 11 9 5 4 6 2 10 7)(3 8)