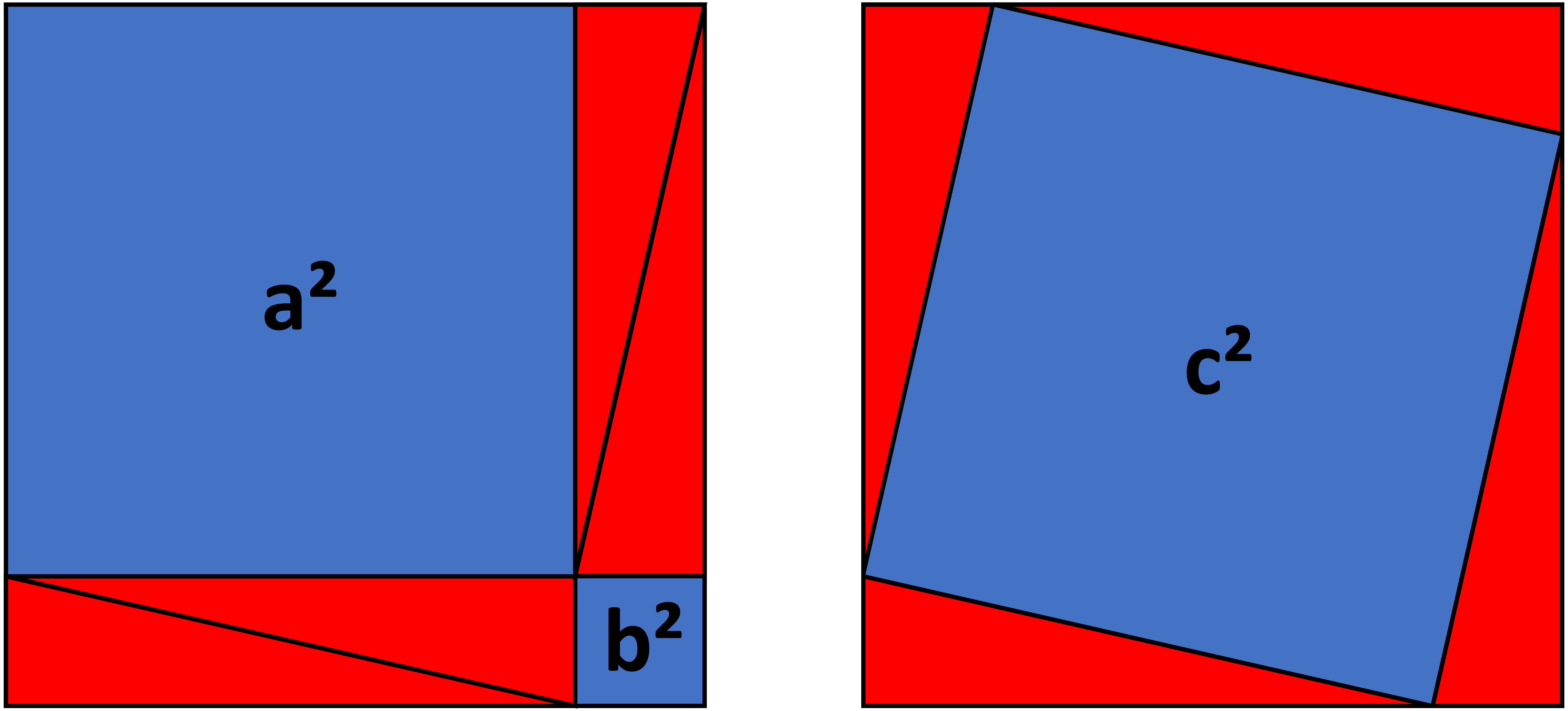
Pleasing Pictorial Proofs and Ptolemy's Ptheorem

Martin Harris

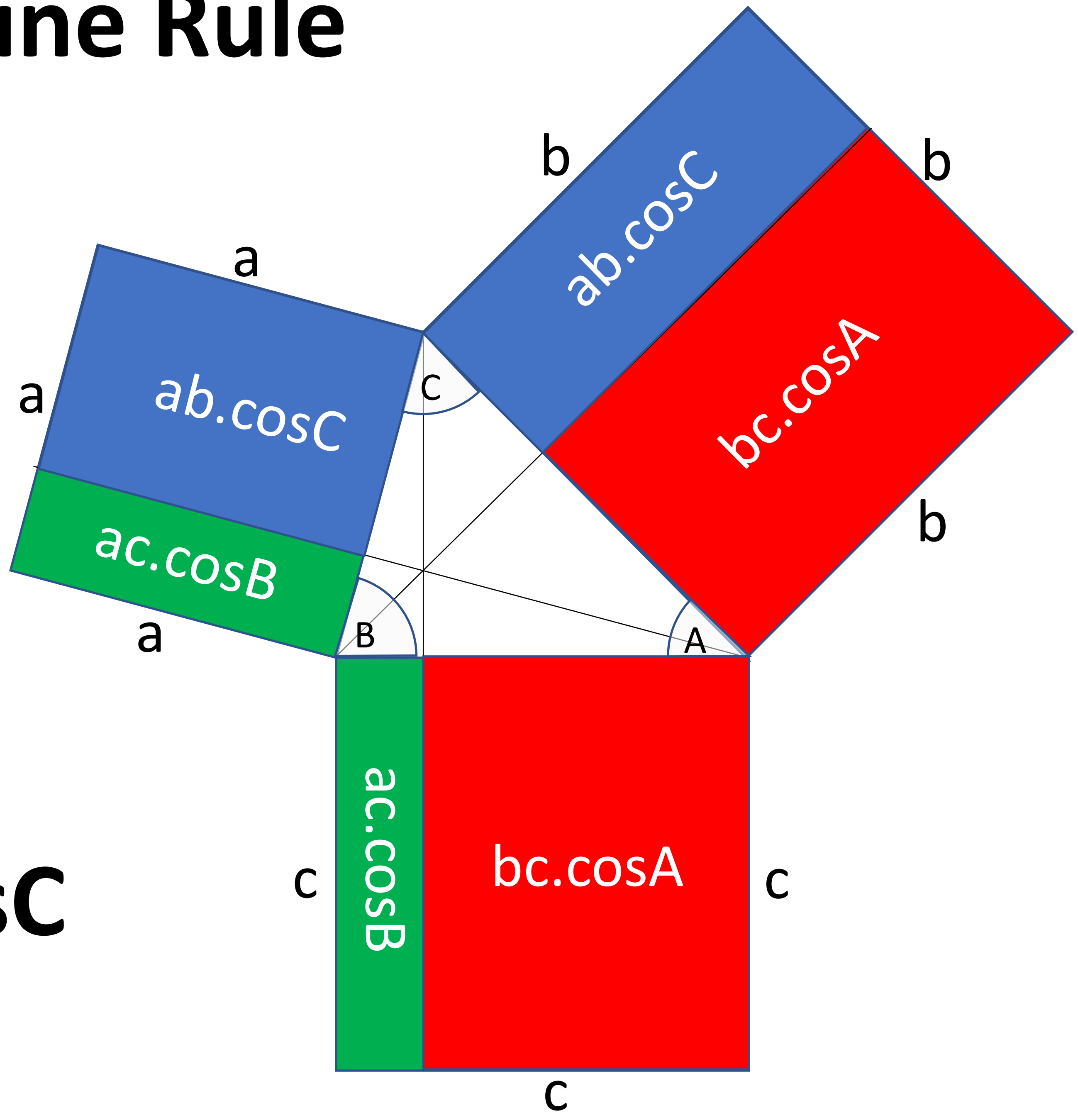
@MarHarStar

MathsJam Annual Conference 2018

Pictorial Proof of Pythagoras



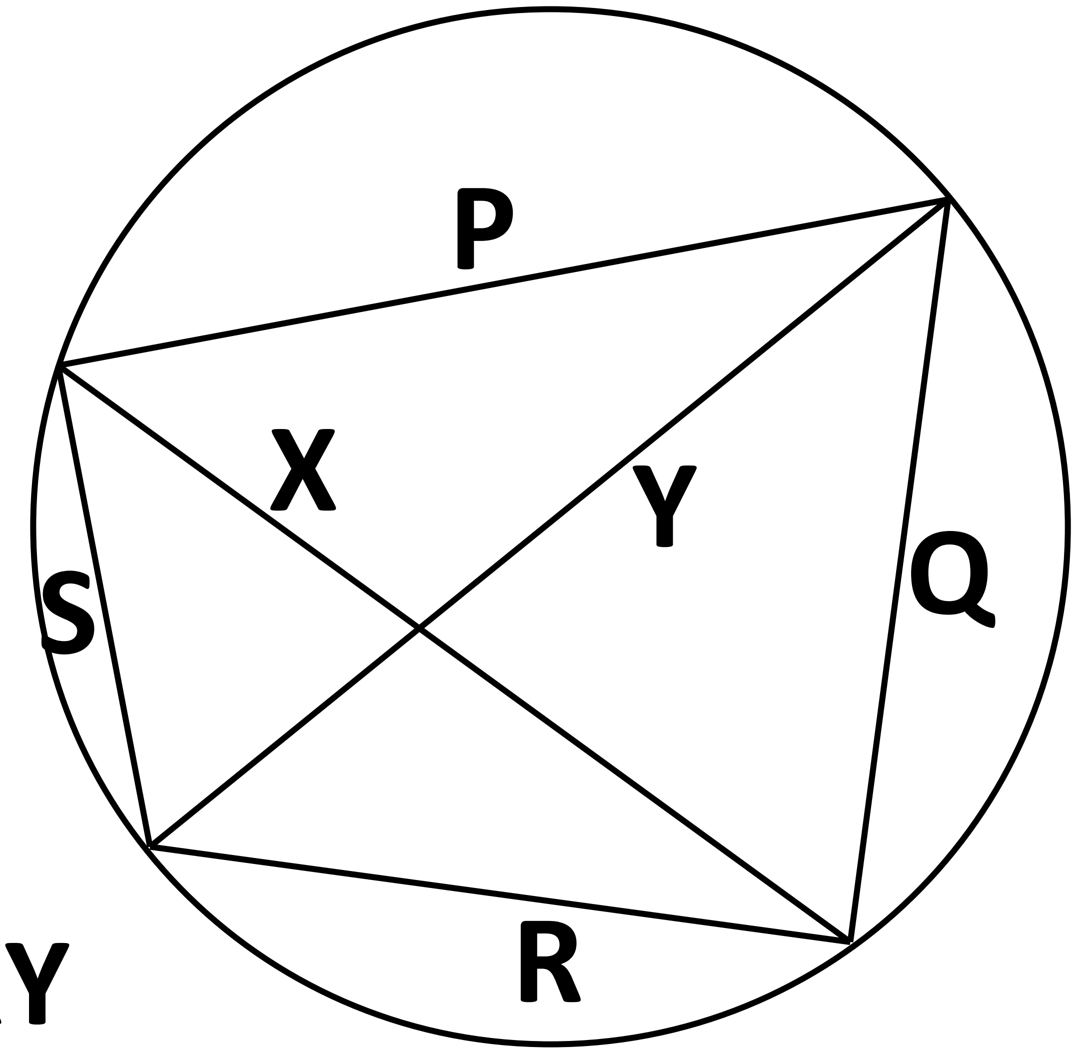
Pictorial Proof of Cosine Rule



$$c^2=a^2+b^2-2ab.\cos C$$

Ptolemy's Theorem:

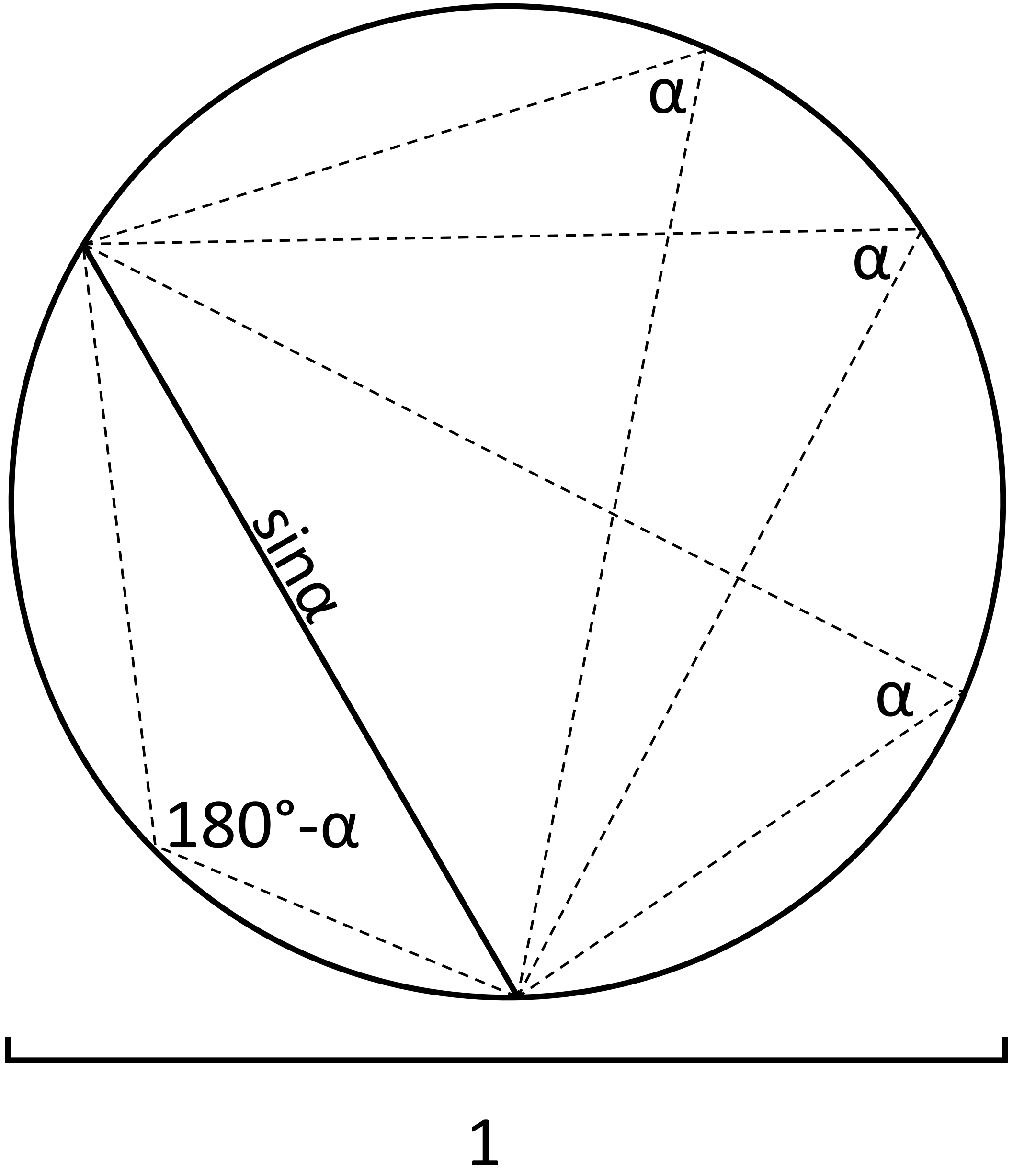
The product of the diagonals of a cyclic quadrilateral is equal to the sum of the products of opposite pairs of sides.



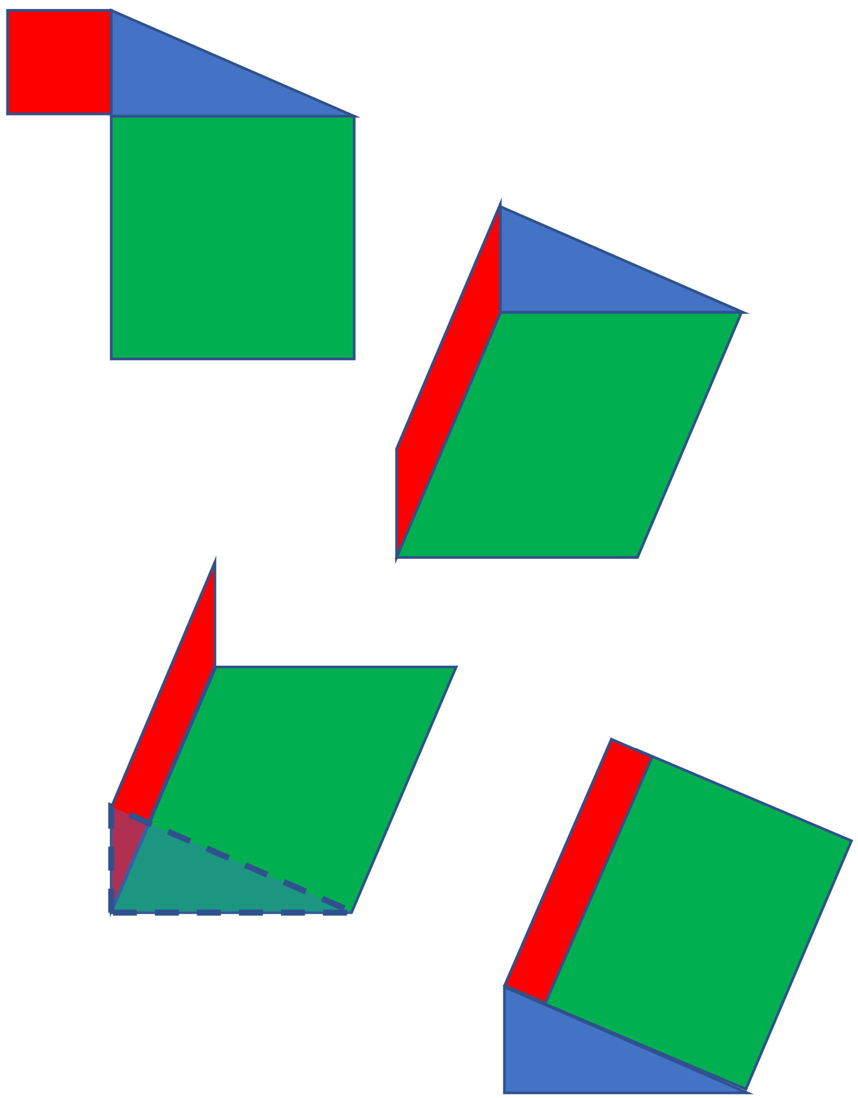
PR + QS = XY

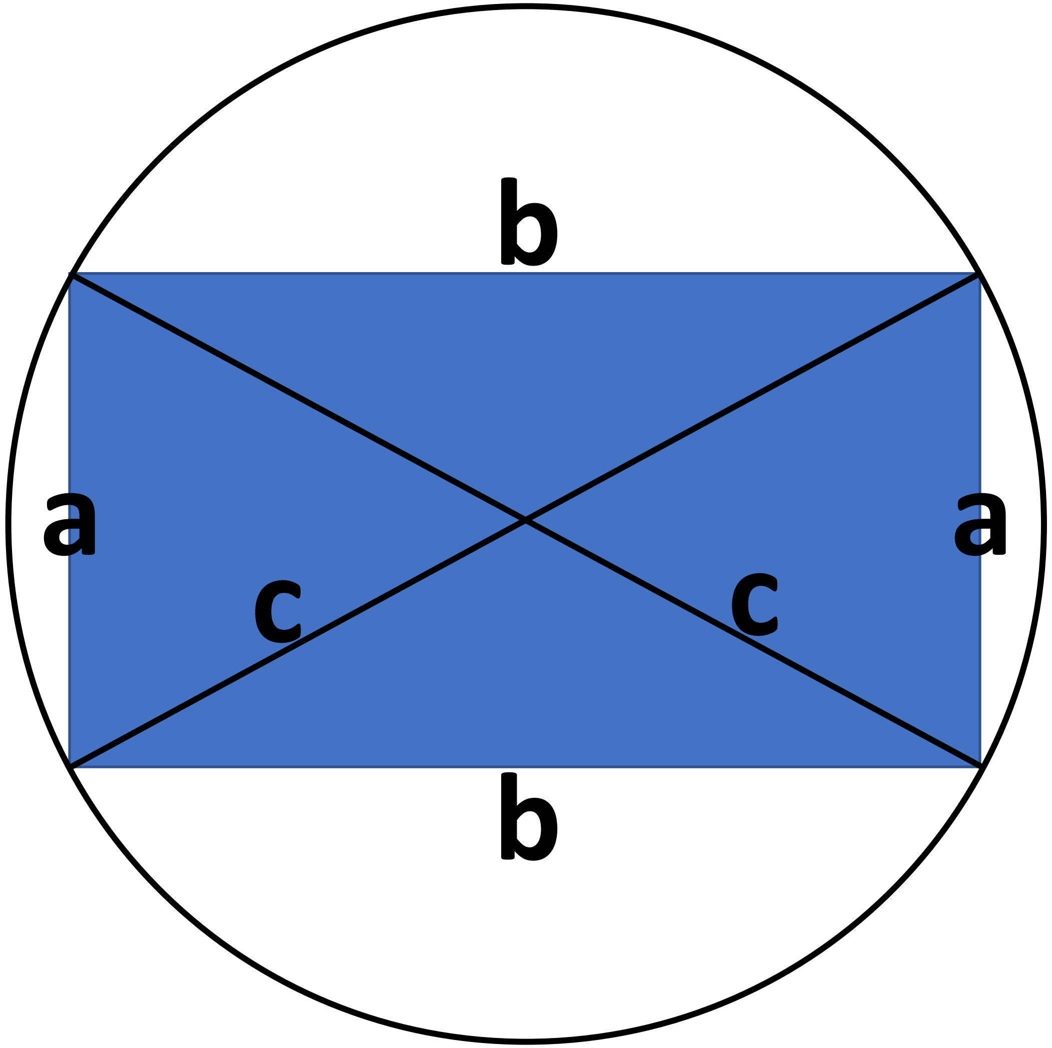
Refresher on Circle Theorems

1) Angles subtended by the same chord are the same angle
2) On a unit diameter circle, the chord subtended by angle α has length sin α
3) Opposite angles add up to 180°

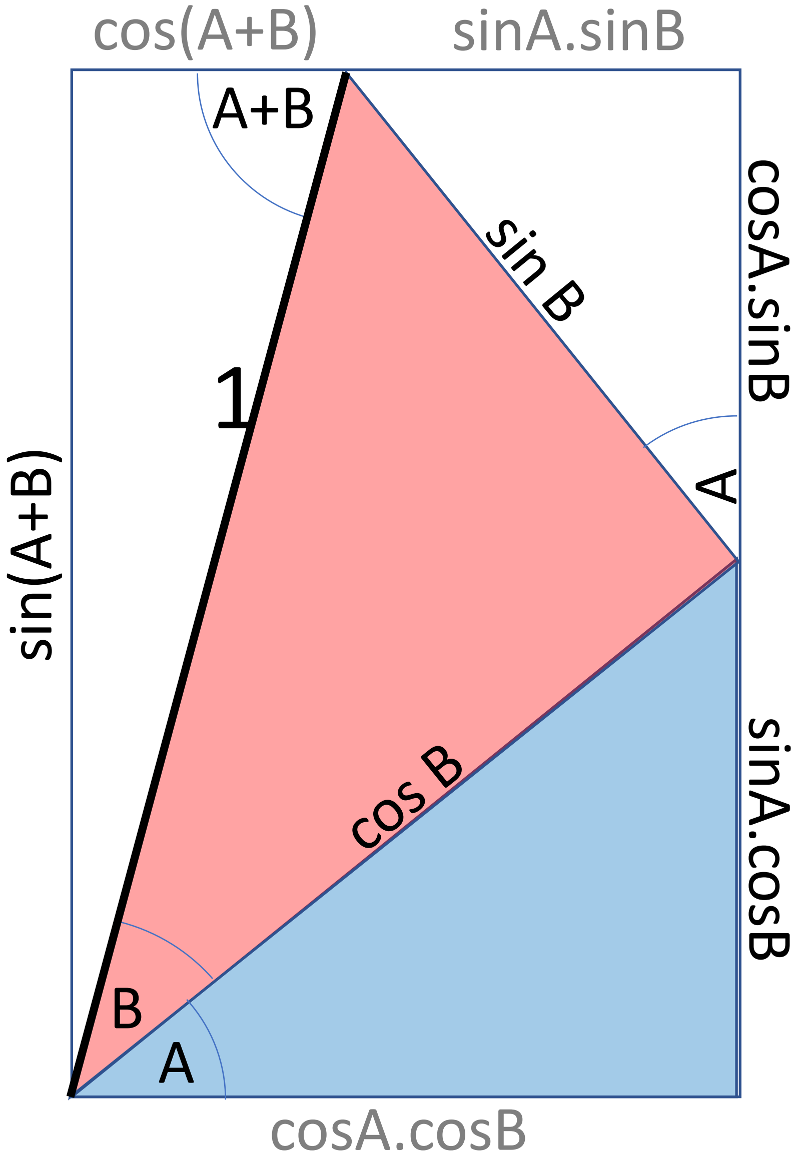


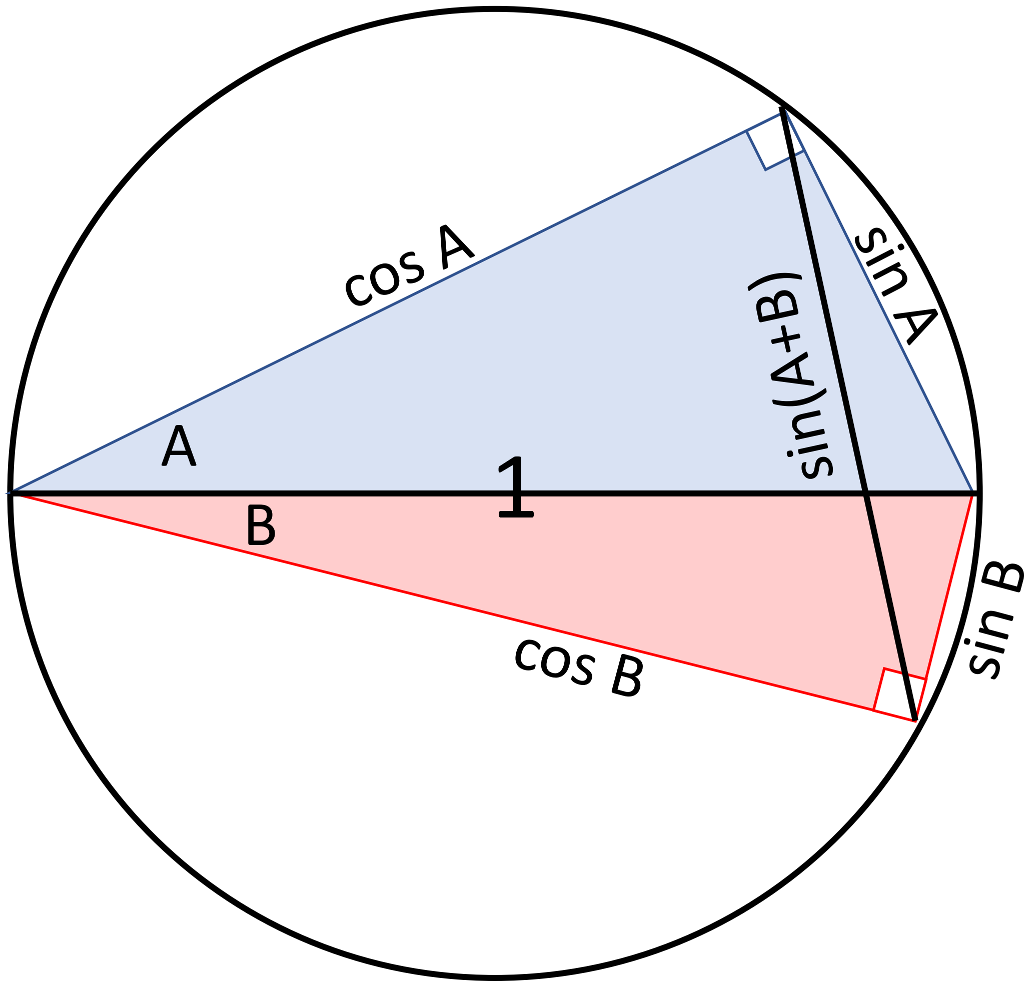
Pythagoras’ Theorem



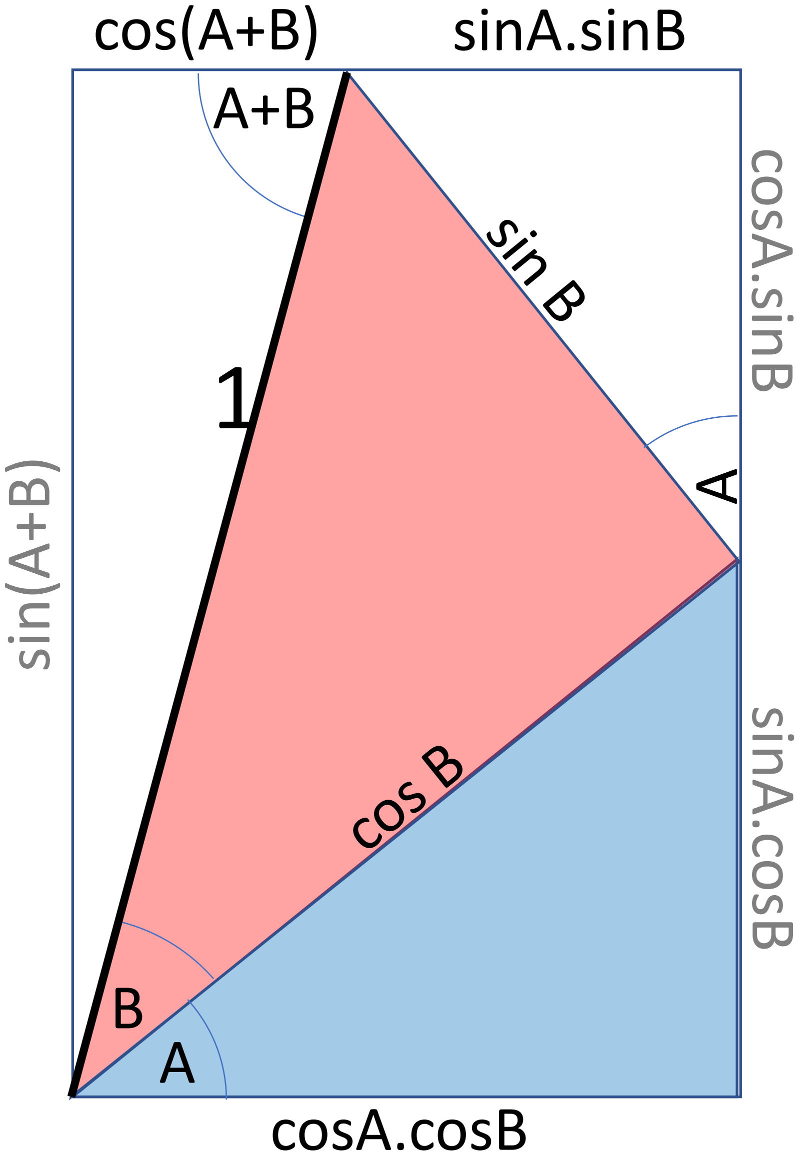


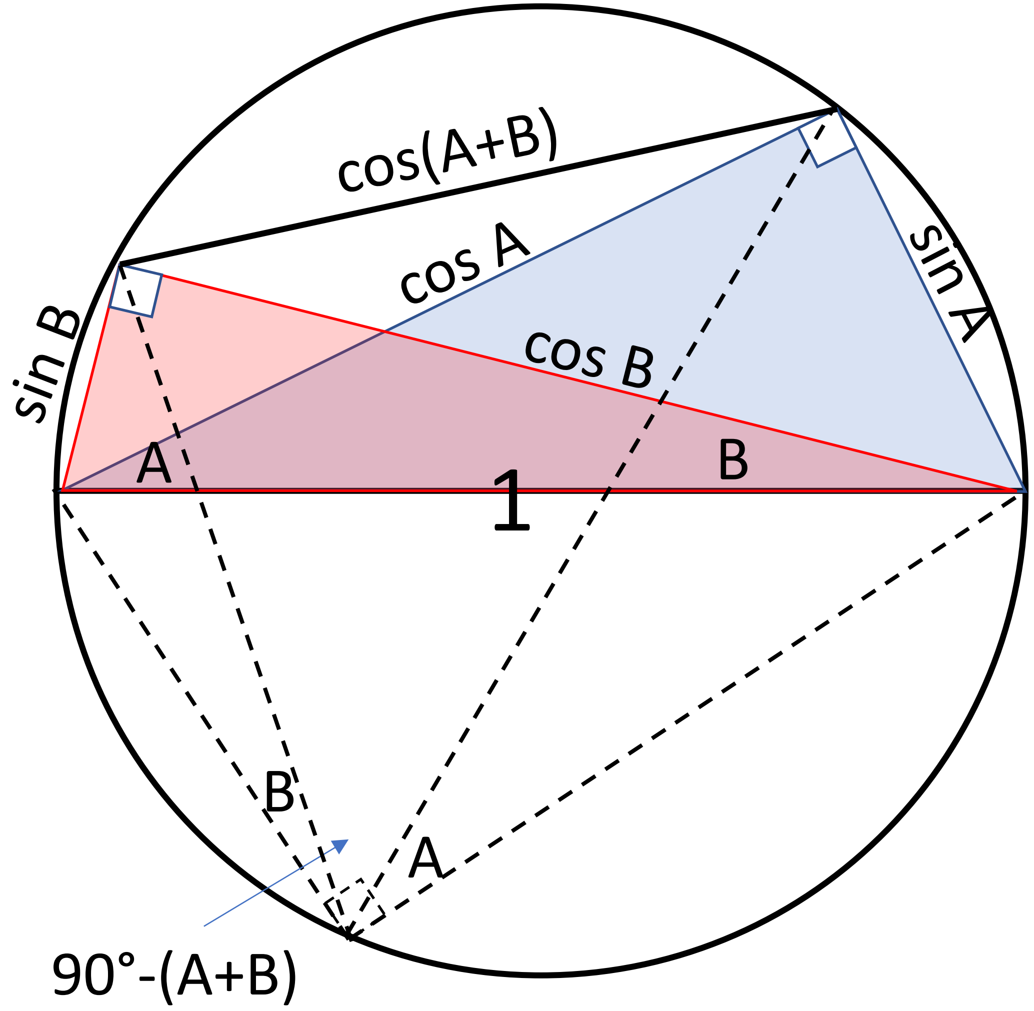
Double Angle Formulae (I)



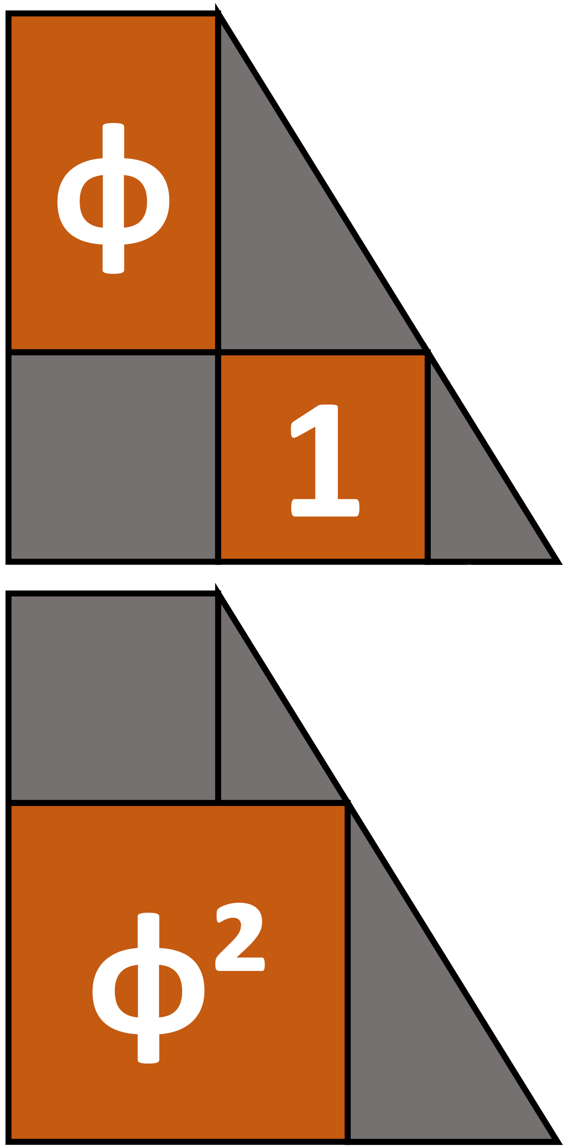


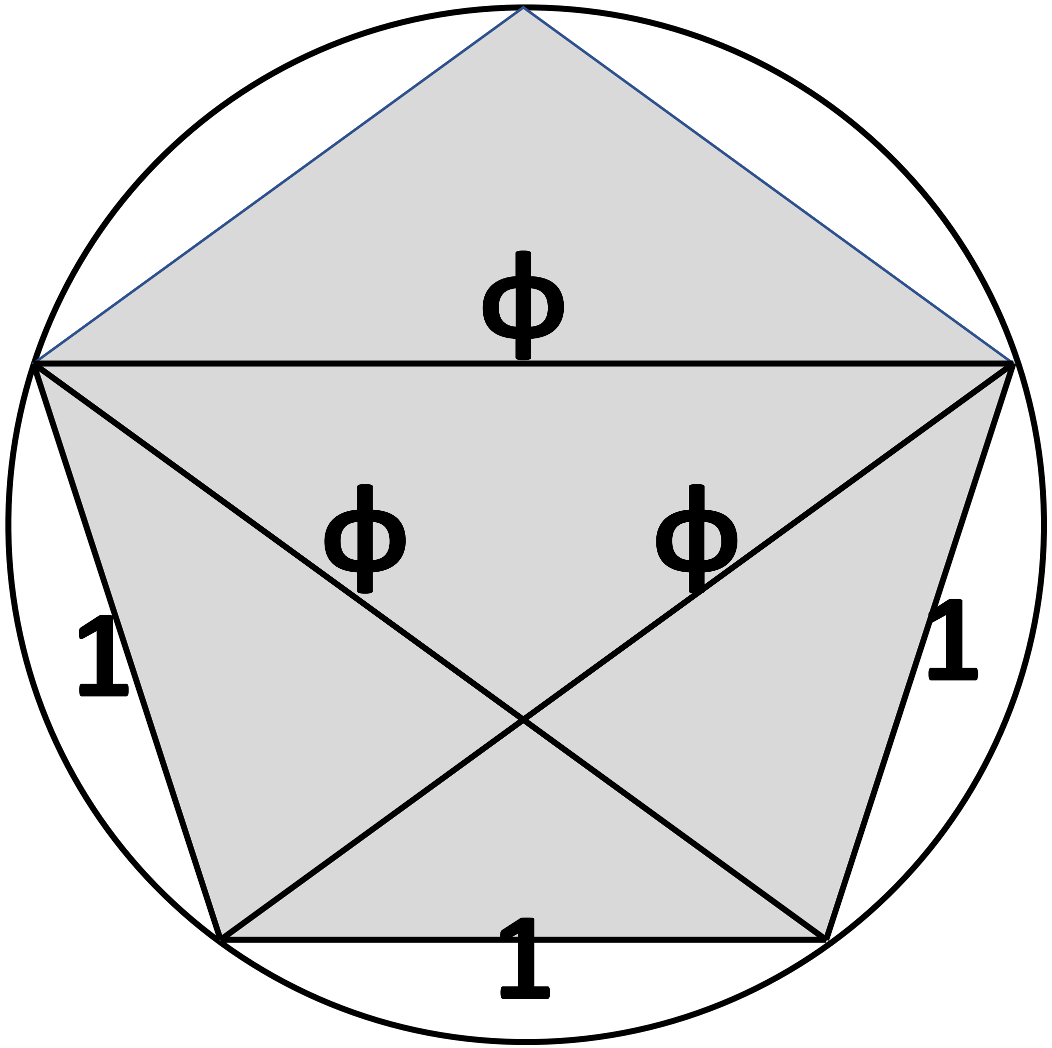
Double Angle Formulae (II)





Golden Ratio: $\phi^2 = \phi + 1$

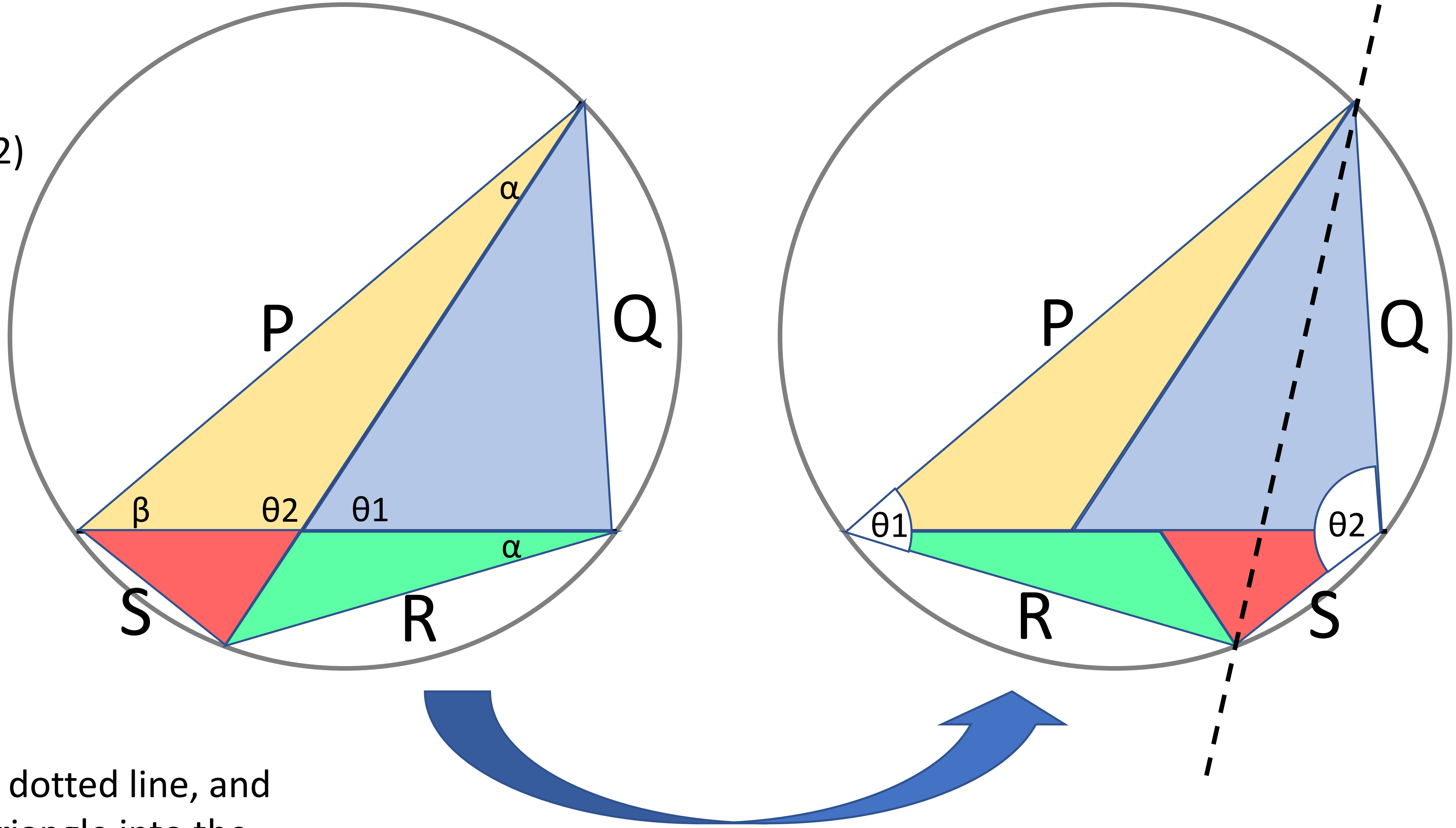




My General Proof

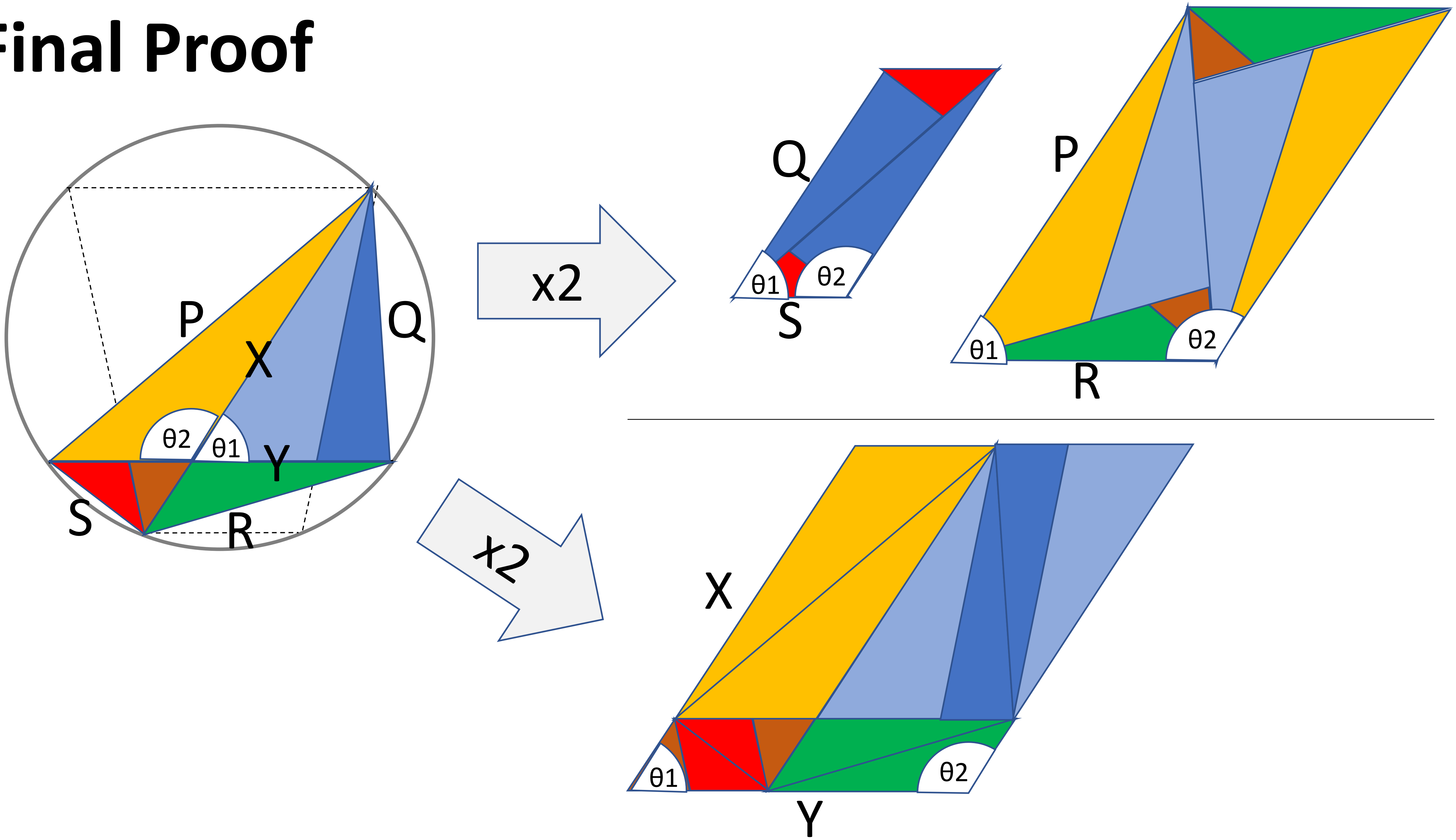
α+β=θ1 (both are 180° - θ2)

If we flip the bottom section, the new quadrilateral has previously opposite sides now adjacent, with θ angles between them



We can then cut along the dotted line, and make two copies of each triangle into the required parallelograms

Final Proof



Thank You

Desmos:

https://www.desmos.com/calculator/6ebufxblaa

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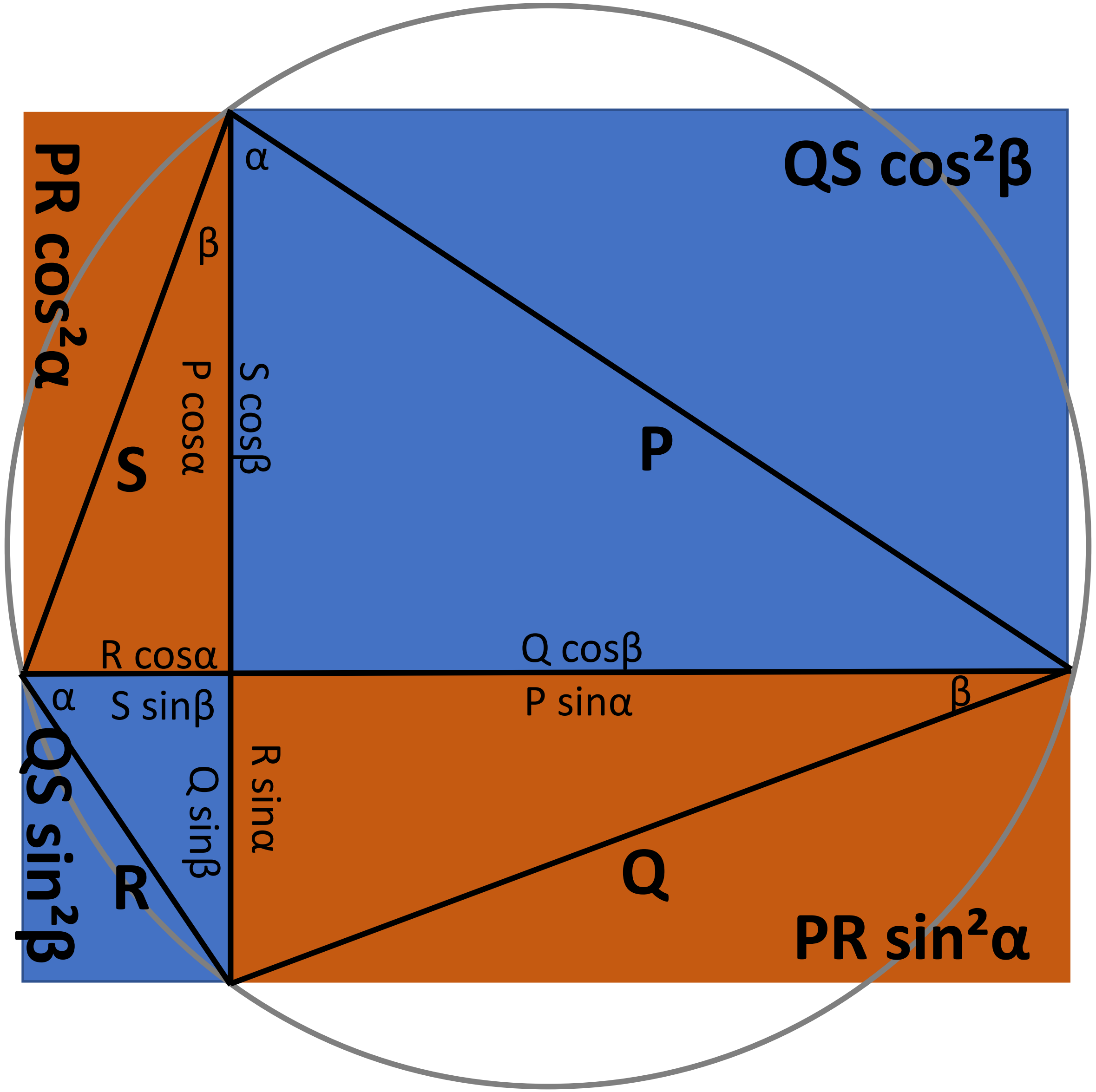
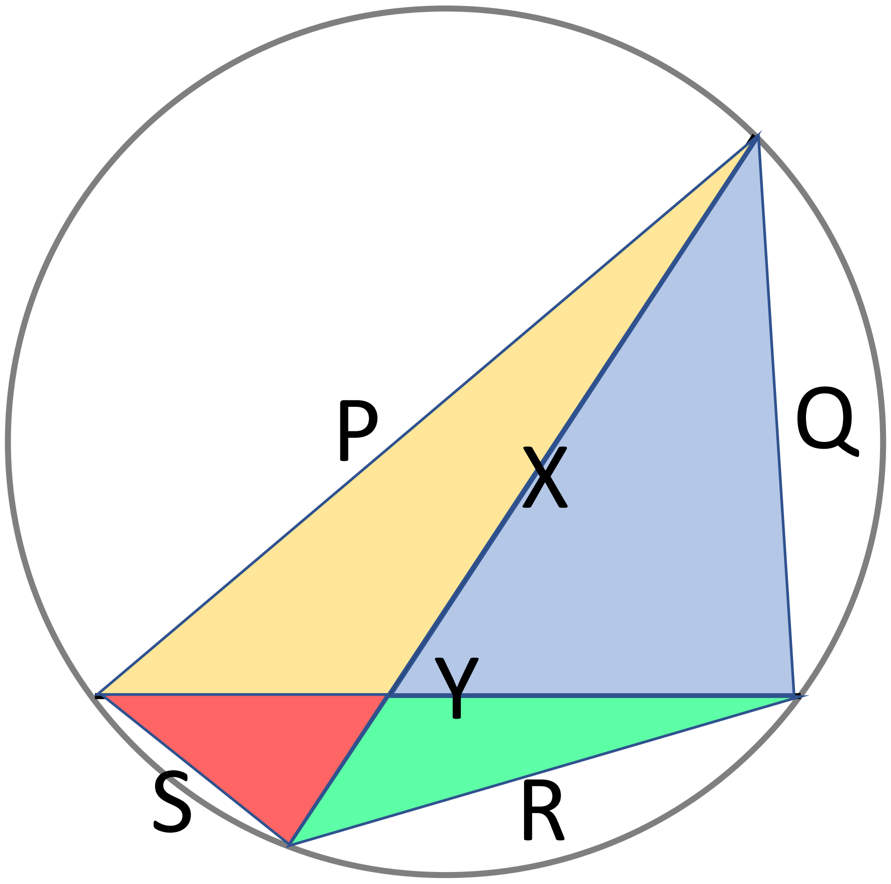
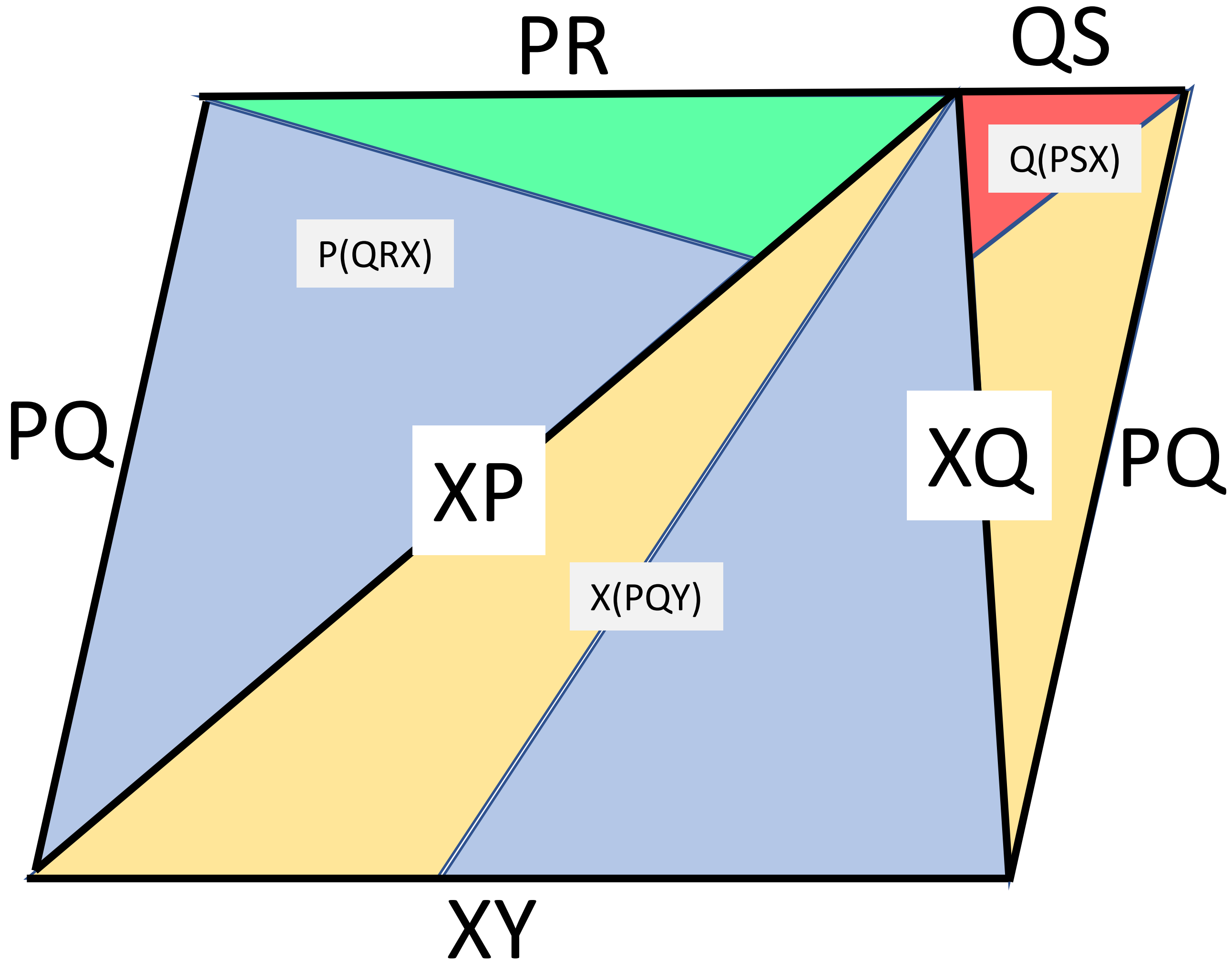
A Nice Special Case



An Unsatisfying (but Technically Correct) Proof





Line lengths are not scalar factors!