n	1	2	3	4	5
$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$	$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$	$\begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

Algorithm

1 → 2 → 3 → 5

$\Theta(n)$

Exponentiation

by squaring: $\Theta(\log n)$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = P \begin{pmatrix} \left(\frac{1-\sqrt{5}}{2}\right)^n & 0 \\ 0 & \left(\frac{1+\sqrt{5}}{2}\right)^n \end{pmatrix} P^{-1}$$

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n \right)$$

Divisibility: $m \mid n \Rightarrow F_m \mid F_n$

Proof: Let $n = km$.

$$\begin{pmatrix} F_{m+1} & F_m \\ F_m & F_{m-1} \end{pmatrix}$$ ← Diag modulo F_m

So is its k^{th} power,

$$\begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{km} = \begin{pmatrix} F_{m+1} & F_m \\ F_m & F_{m-1} \end{pmatrix}^k$$