Everything Old is New Again

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(I help them commit it)

Someone has to tell you these at some point.

Simpson's Paradox?

Someone has to tell you these at some point.

Bertrand's Paradox?

Someone has to tell you these at some point.

The Heavy Boots story?

Someone has to tell you these at some point.

The Barometer/Manometer story?

Someone has to tell you these at some point.

The “... plus a constant” joke?

Someone has to tell you these at some point.

Countability of algebraic numbers?

Someone has to tell you these at some point.

That thing with the right angles?

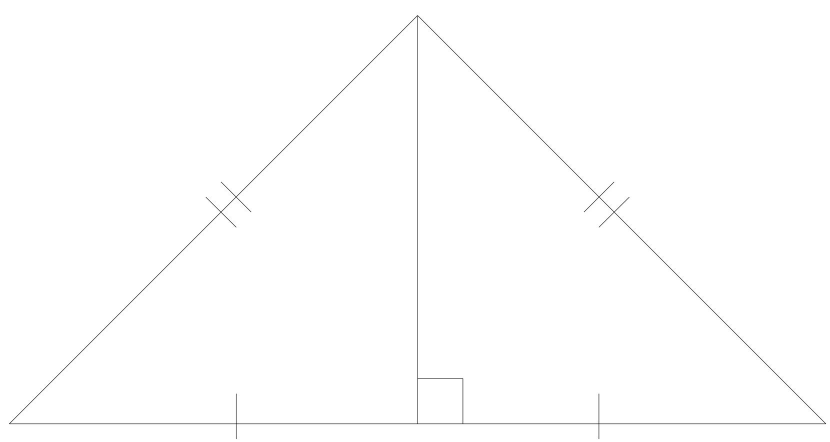
An Apology

If you've seen this before then I'm sorry. Have a cup of tea and/or a biscuit. I know from past MathsJams that some people haven't seen this, so thought it was worth using my slot up to try to remedy this.

A Lemma

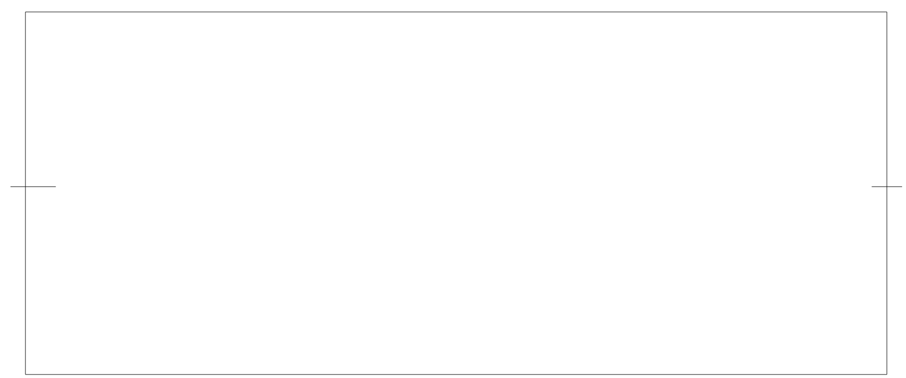
I'll be using this more than once so it seems best to mention it ahead of time.

Given a triangle, if the perpendicular bisector of the base passes through the opposite vertex, then the triangle is isosceles.



A Rectangle

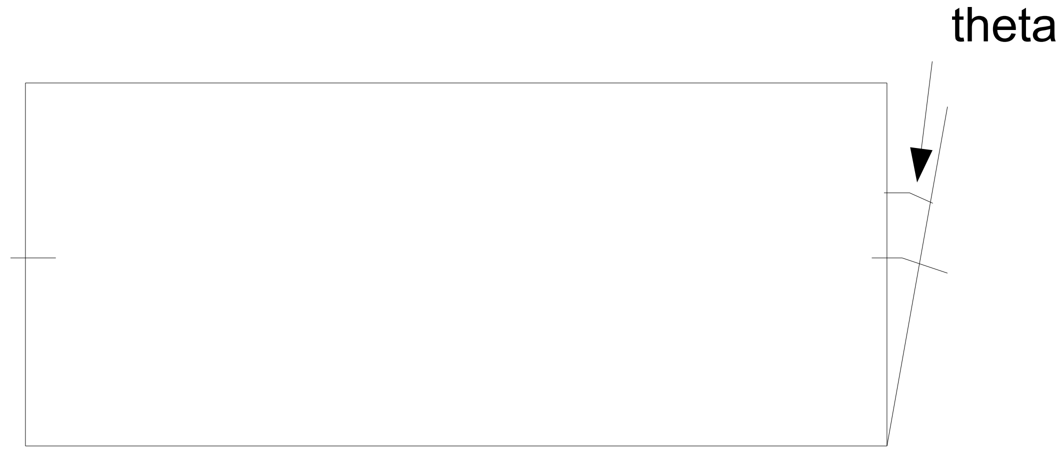
Here is a rectangle. As you may know, opposite sides are of equal lengths, and all the corners are right angles.



In particular, the two marked sides are the same length.

A Rectangle

Let's construct a segment at angle theta to one of the sides, equal in length to that side.



It's the same length as the two marked sides of the rectangle because we've constructed it to be. Theta is not 0 because I say it's not, and this whole exercise would be a bit odd if it were.

Perpendicular Bisector Time

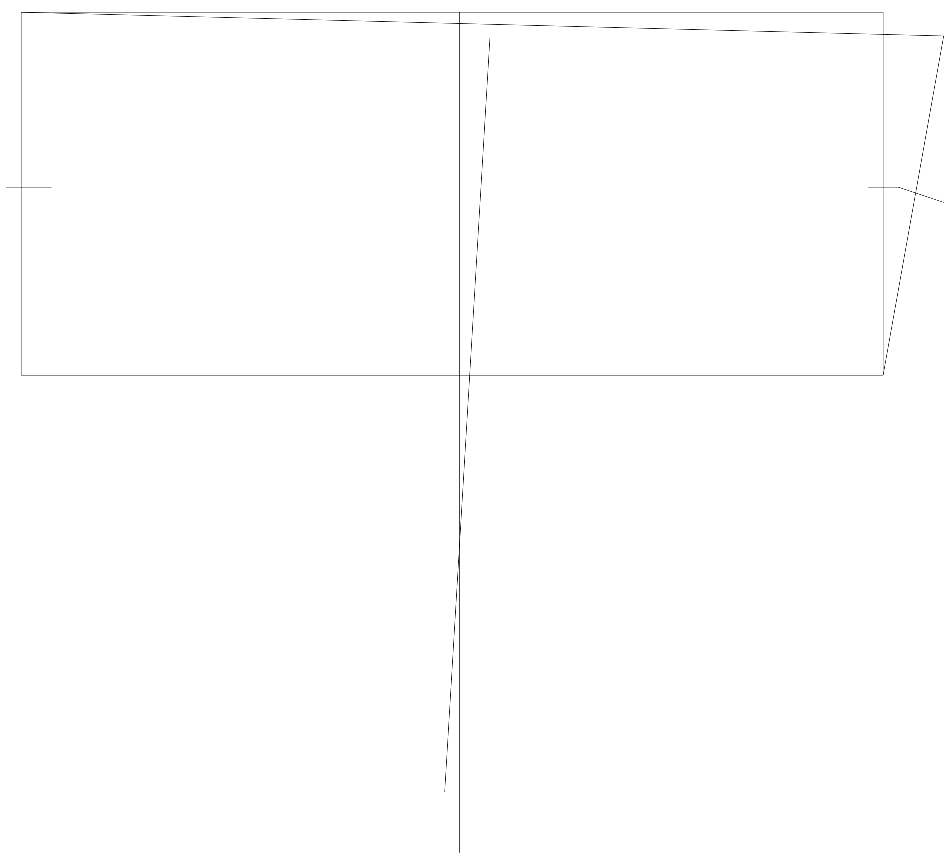
Let's construct the perpendicular bisector of the upper side of the rectangle, heading downwards.

Adding a new line

Join our new point to the opposite upper corner of the rectangle.

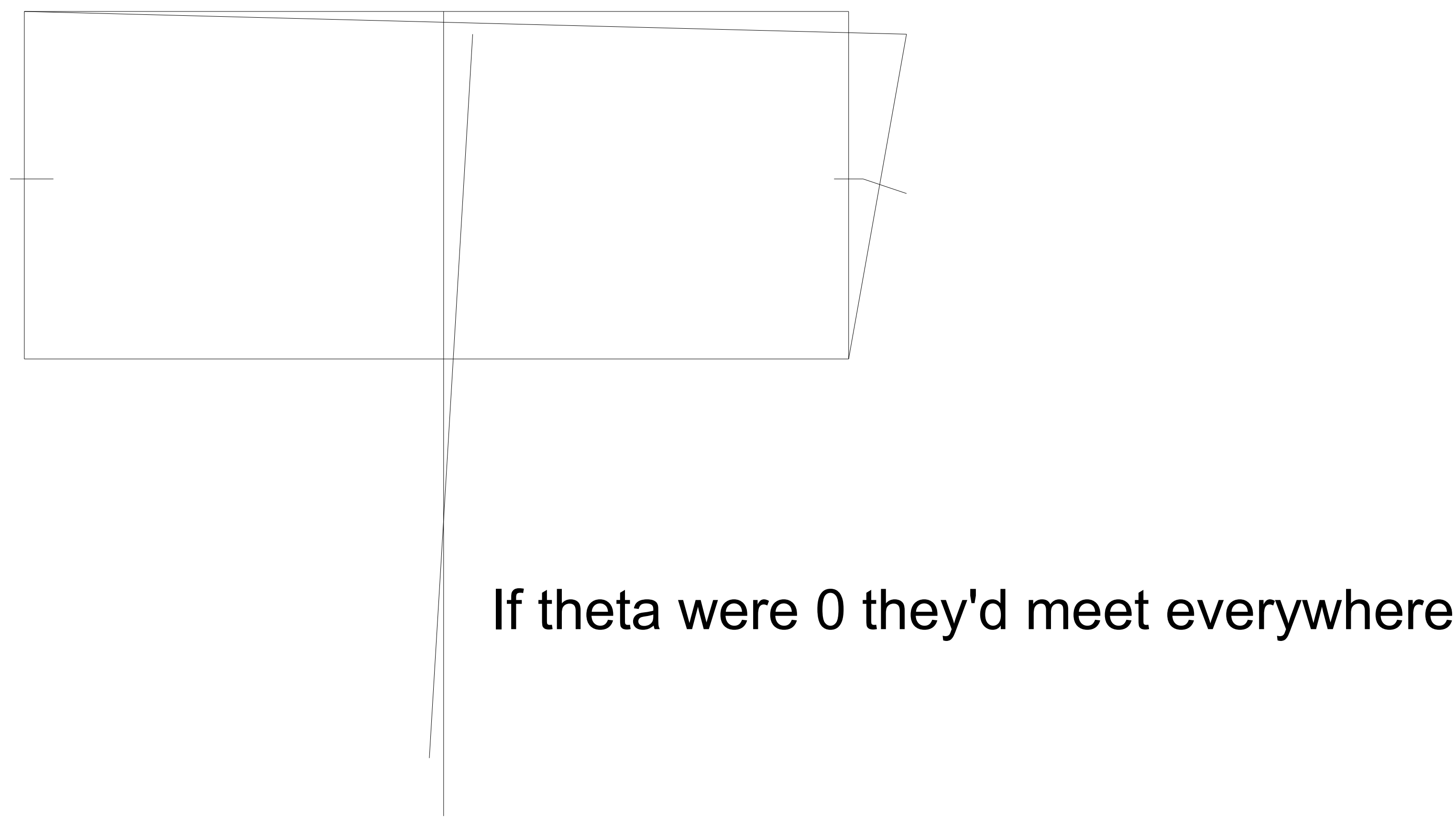
Perpendicular Bisector Time Again

Construct the perpendicular bisector of the new line from the previous slide.



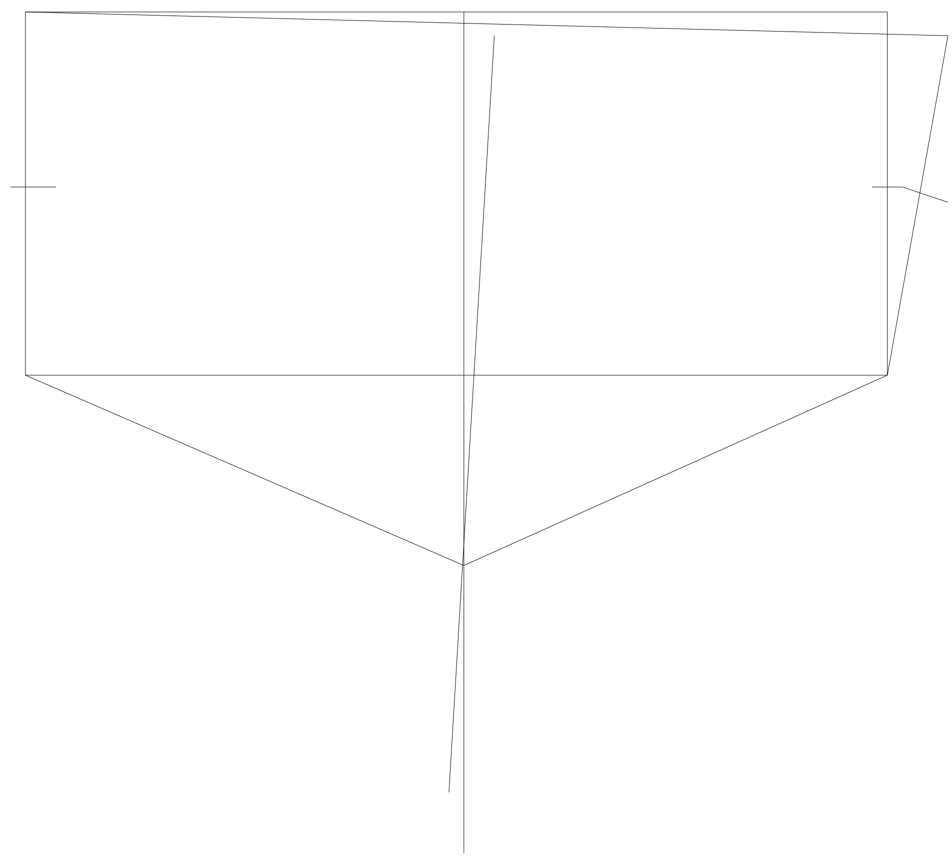
They have to meet somewhere

The two perpendicular bisectors are not parallel (since theta is not 0) so they meet somewhere.



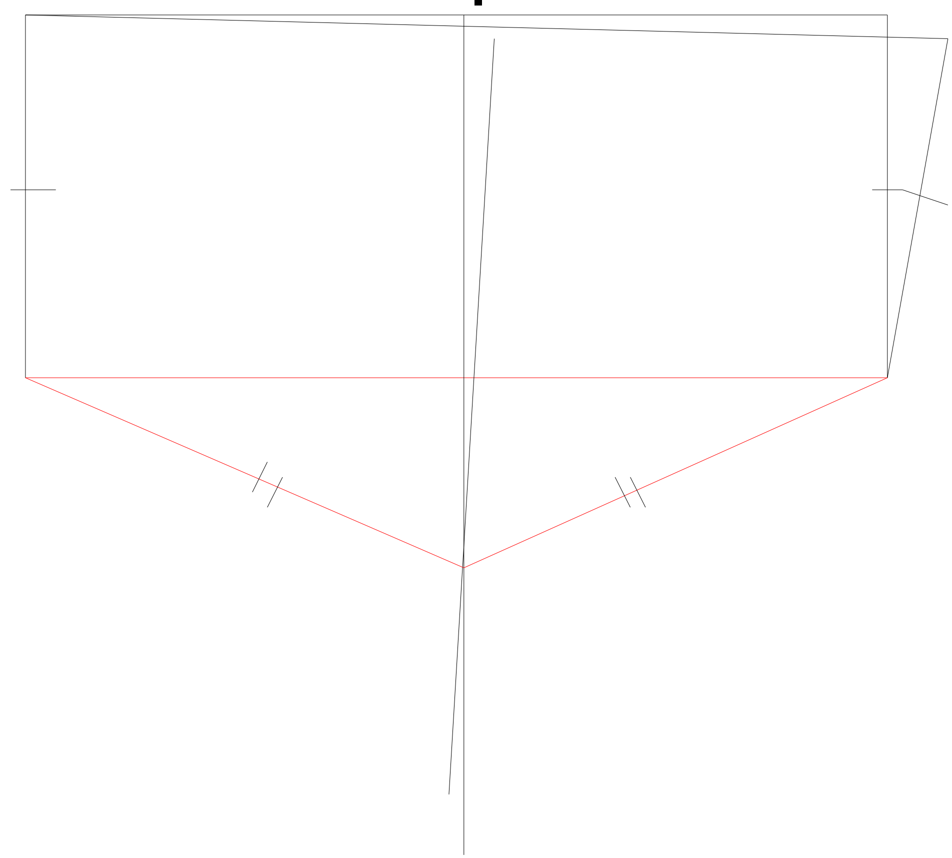
More new lines.

Join the intersection point to the lower two corners of the rectangle.



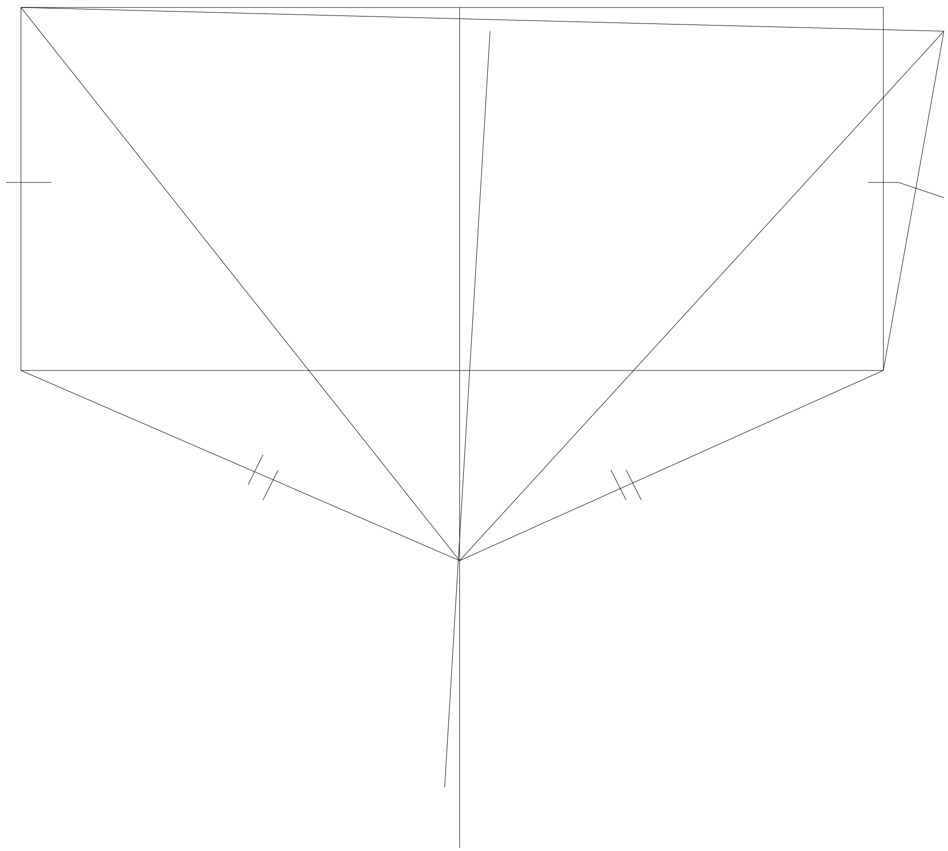
Lemma Time!

The perpendicular bisector of the base of the red triangle at the bottom passes through its vertex. So it is isosceles and the double marked sides are equal.



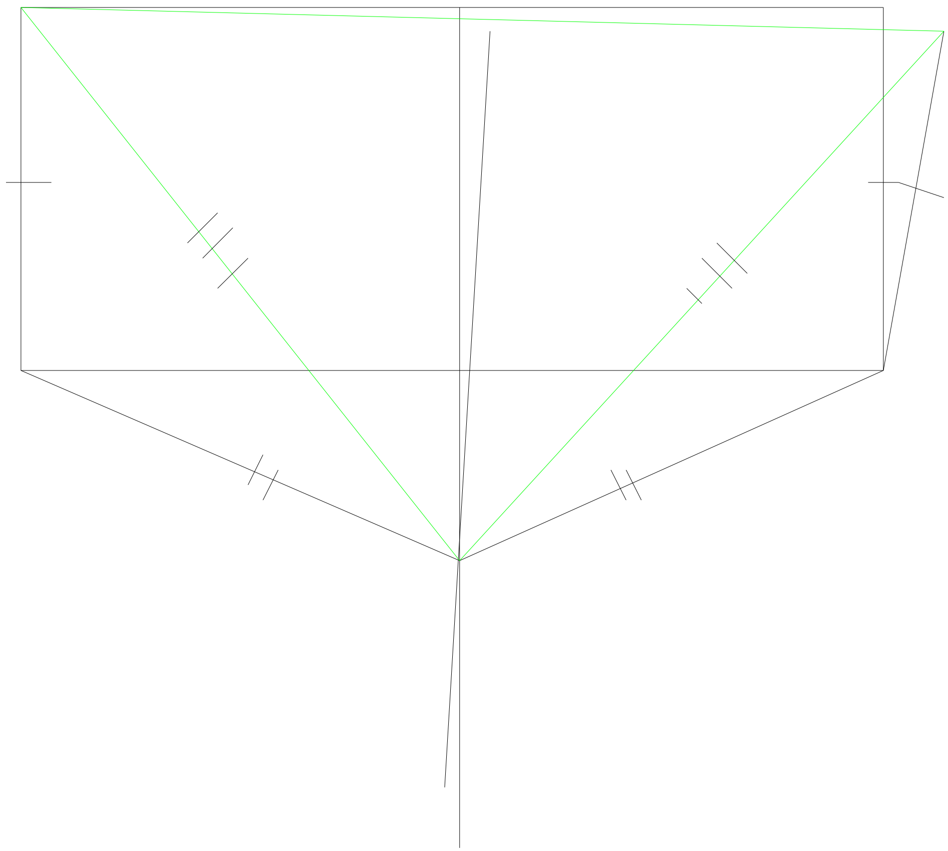
More Lines

Join the intersection point to the top left corner of the rectangle and the point on the end of the slanted line.



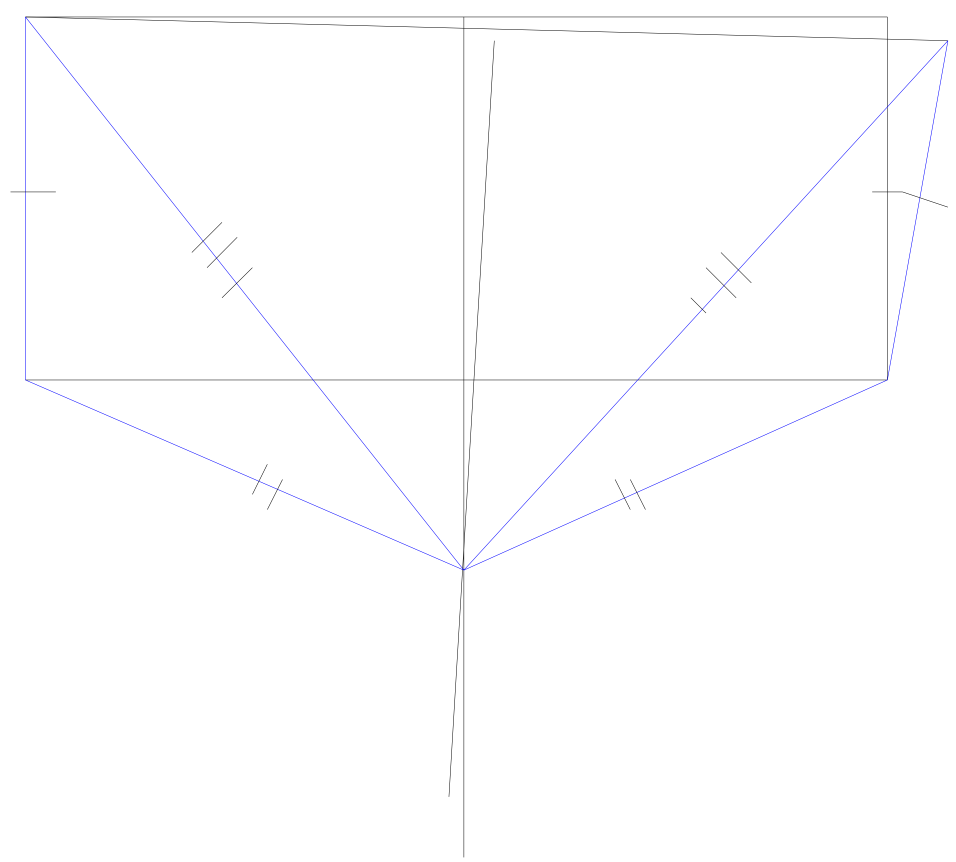
Lemma Time Again

Using our Lemma on the green triangle, the triple marked sides are also equal.



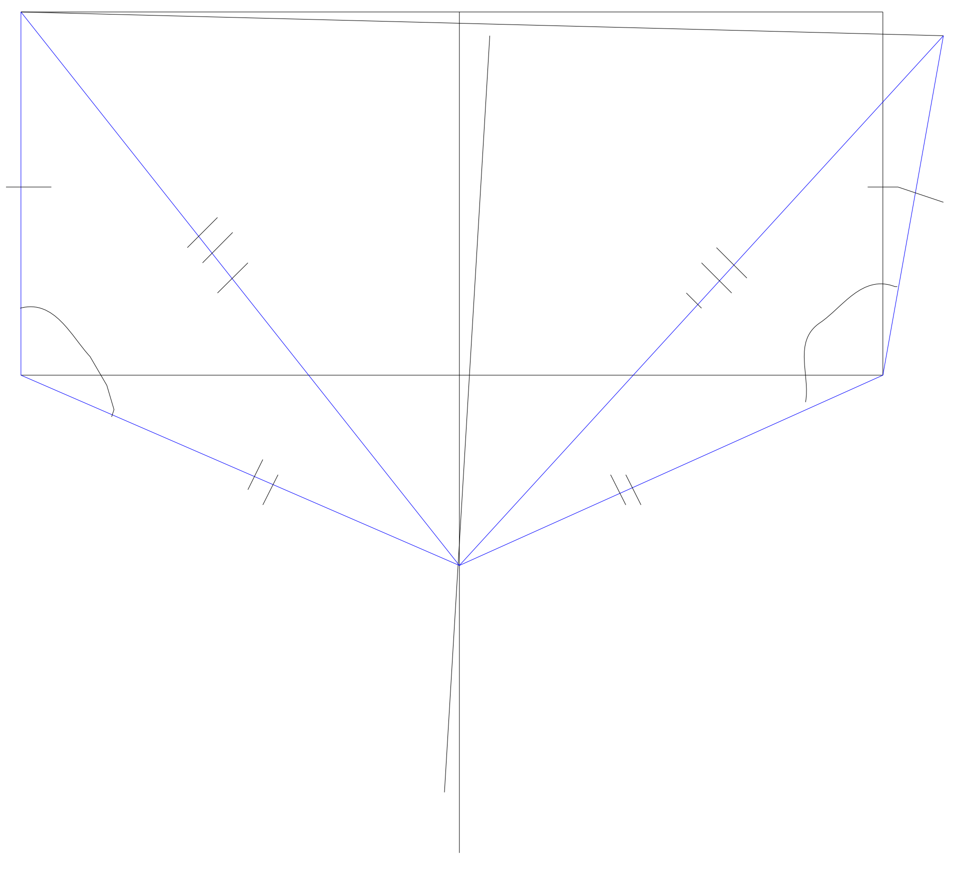
Congruent Triangles (sss)

The two blue triangles have three equal sides so are congruent.



And therefore...

Hence corresponding angles are equal, in particular the marked ones.



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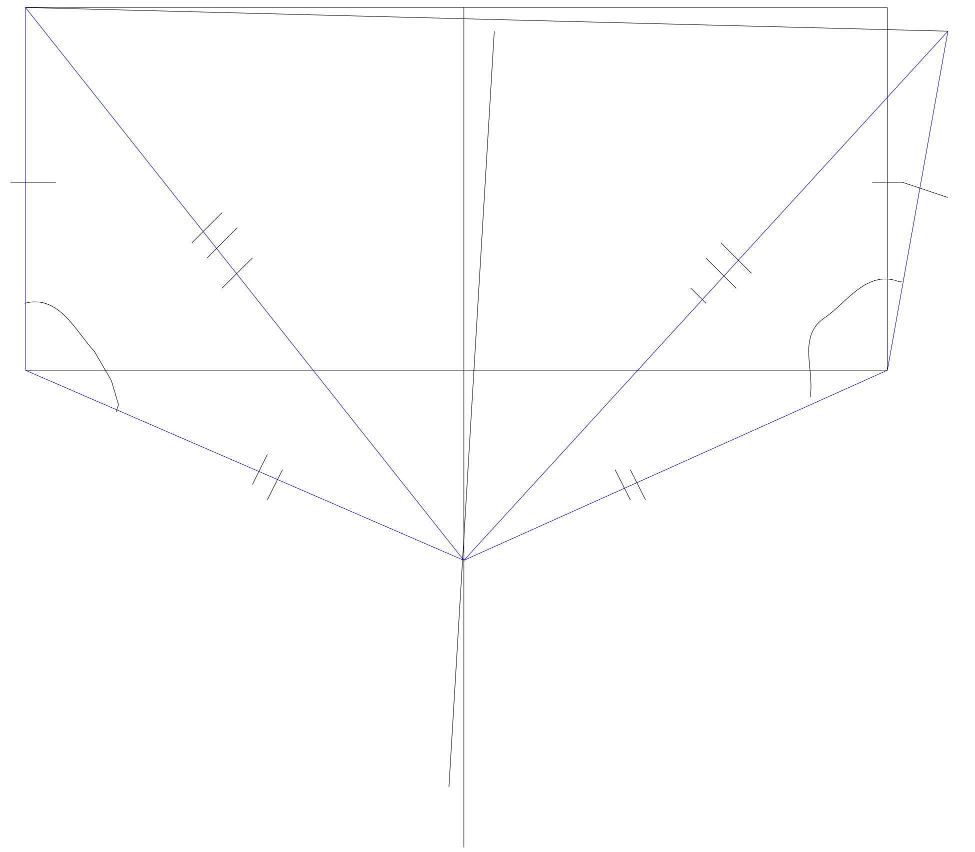
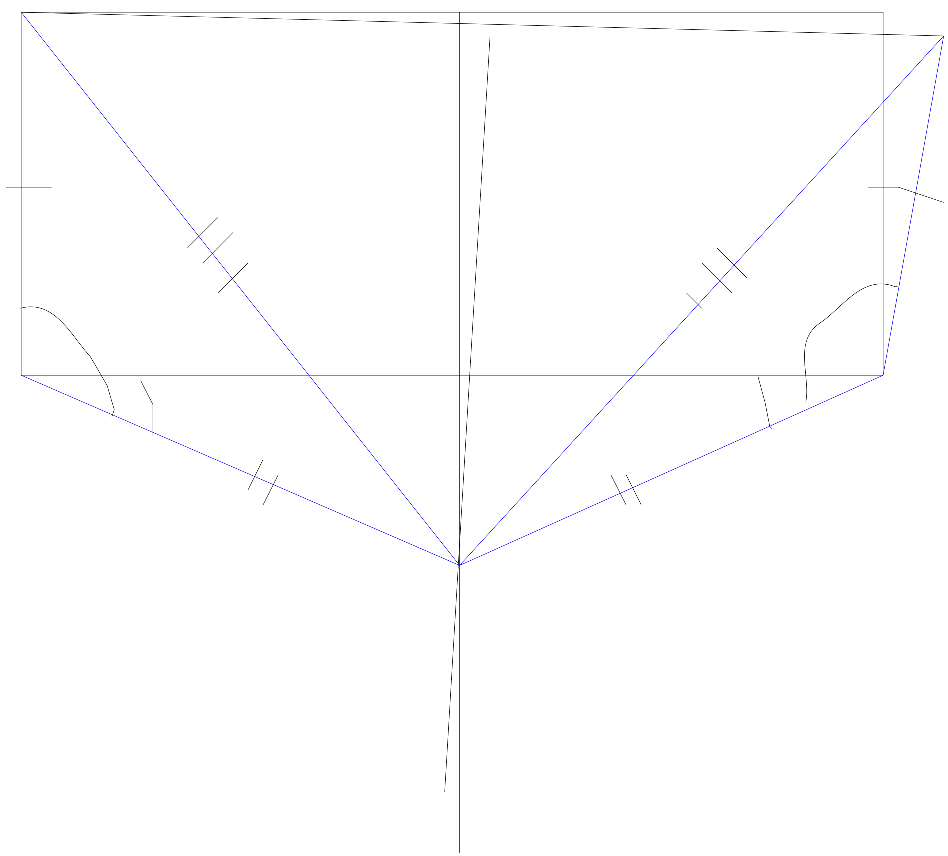


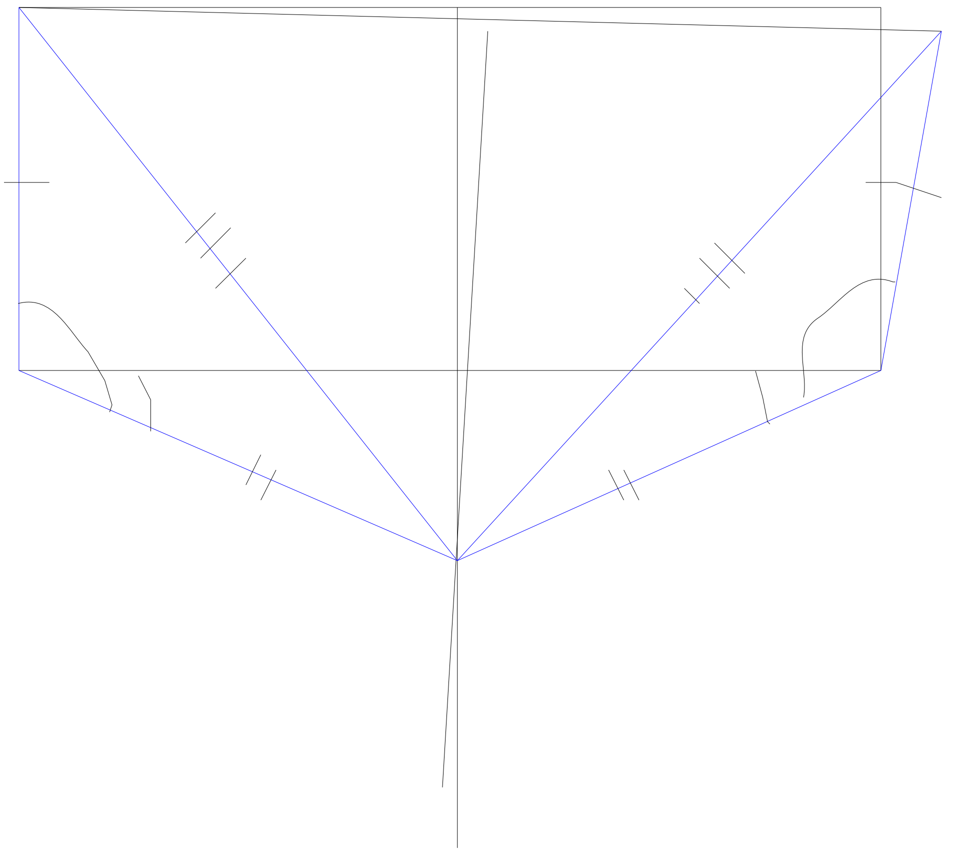
And therefore...

The double marked angles are equal because the triangle at the bottom is isosceles.



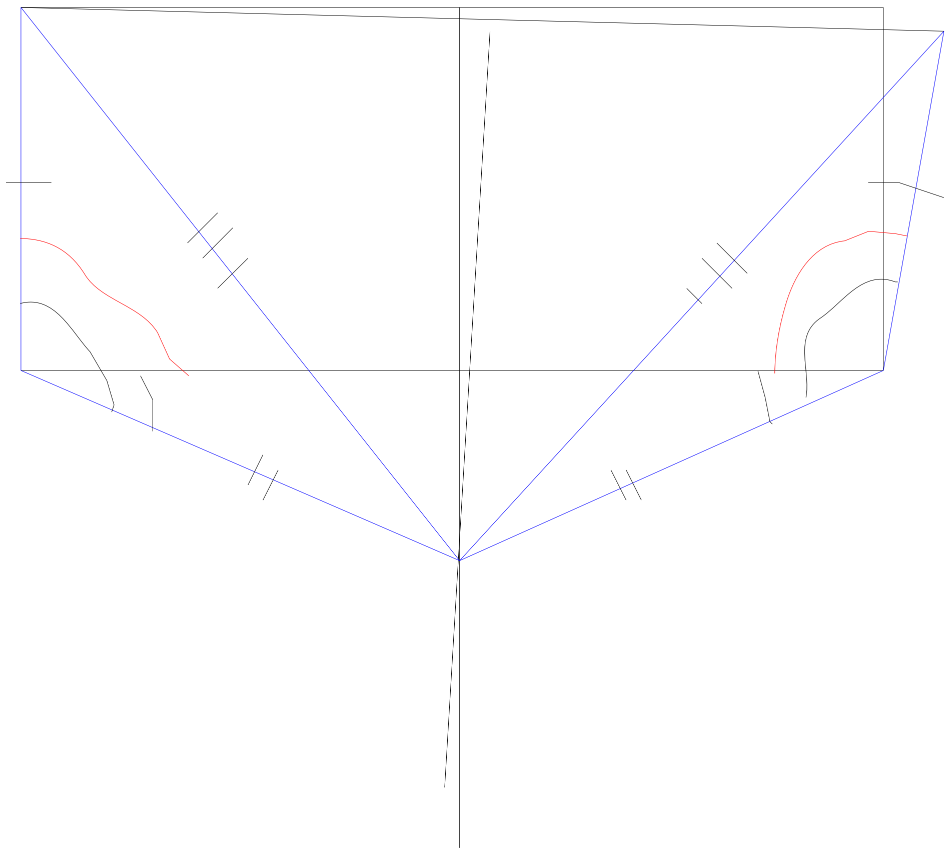
Our conclusion

Subtract the double marked angles from the single marked angles, and we see that...



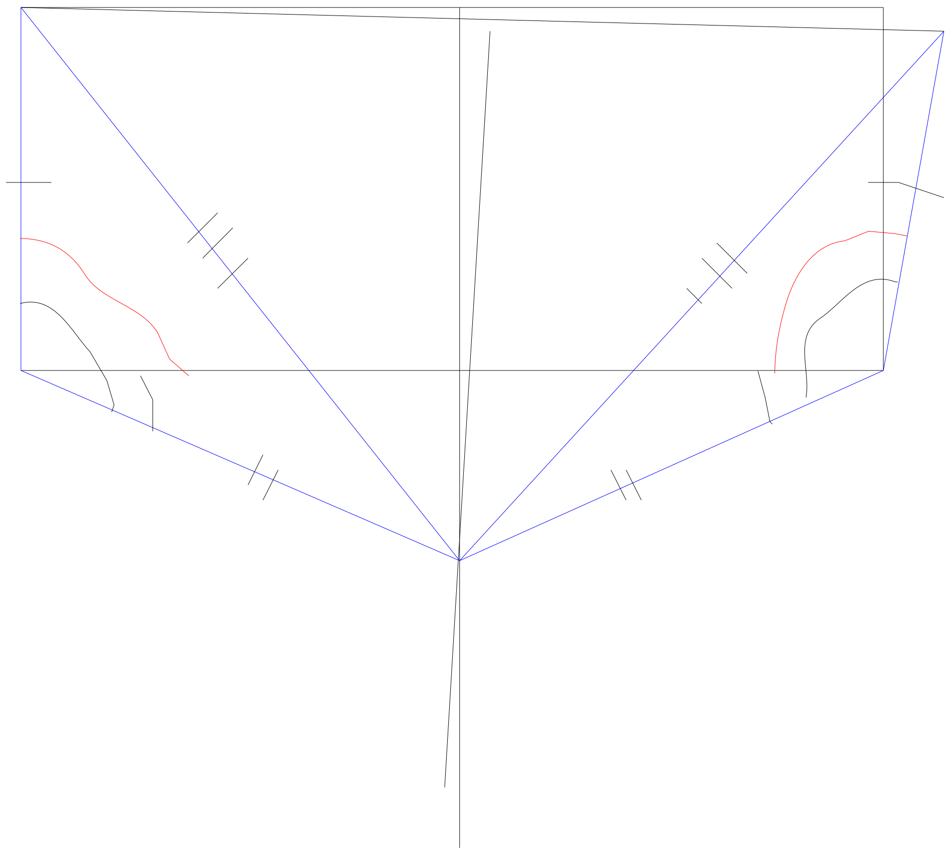
Our conclusion

A right angle is equal to a right angle plus theta. (Angles marked in red.)



Let's address a couple of objections

Maybe the intersection point is elsewhere...



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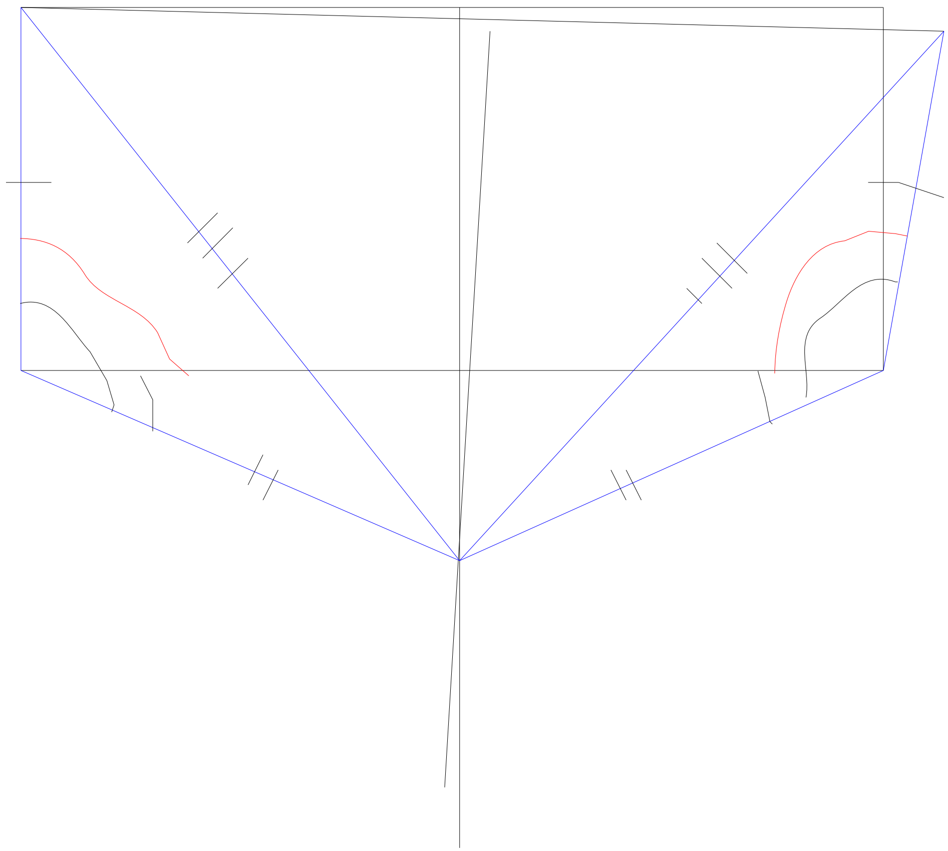
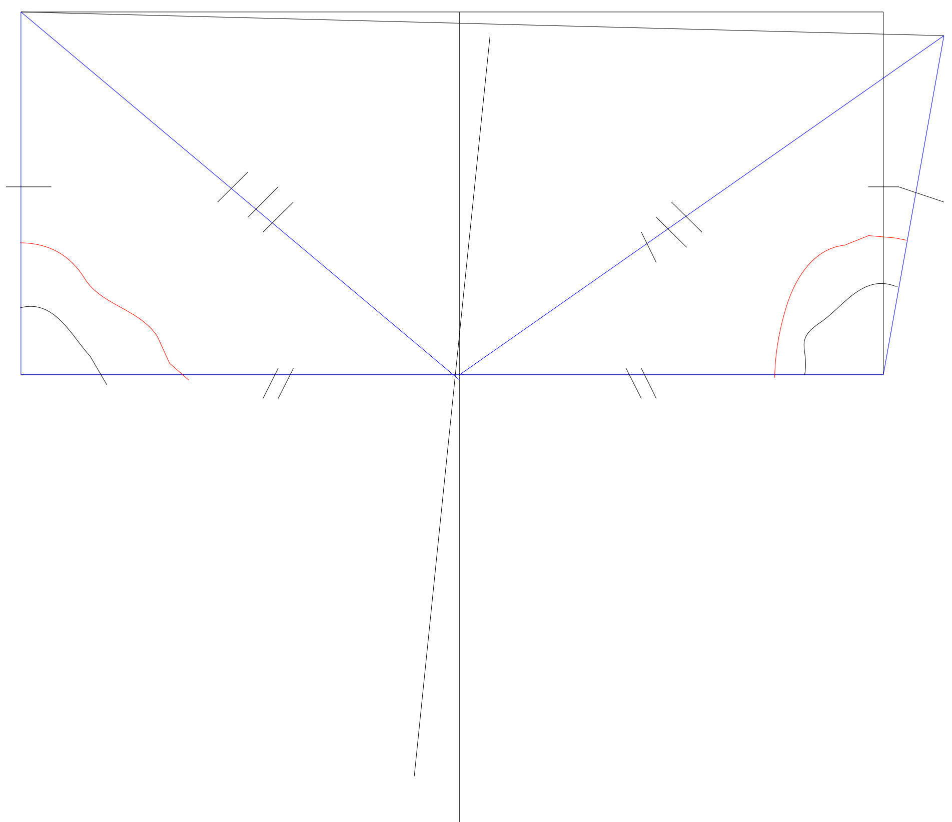


What if point is on edge?

Don't have isosceles bits to remove, so no problem



OK... inside the rectangle?

In this case we add the isosceles bits instead of subtracting them.