One Three hundred and Eighty Fourth

1/384

John Read
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Peter Rowlett @peterrowlett 4 Oct

In this morning's lecture I found the derivative of x^2. That is all.

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@peterrowlett to make a square progressively bigger while remaining square add a sliver on two sides. So the derivative of x^2 is length 2x

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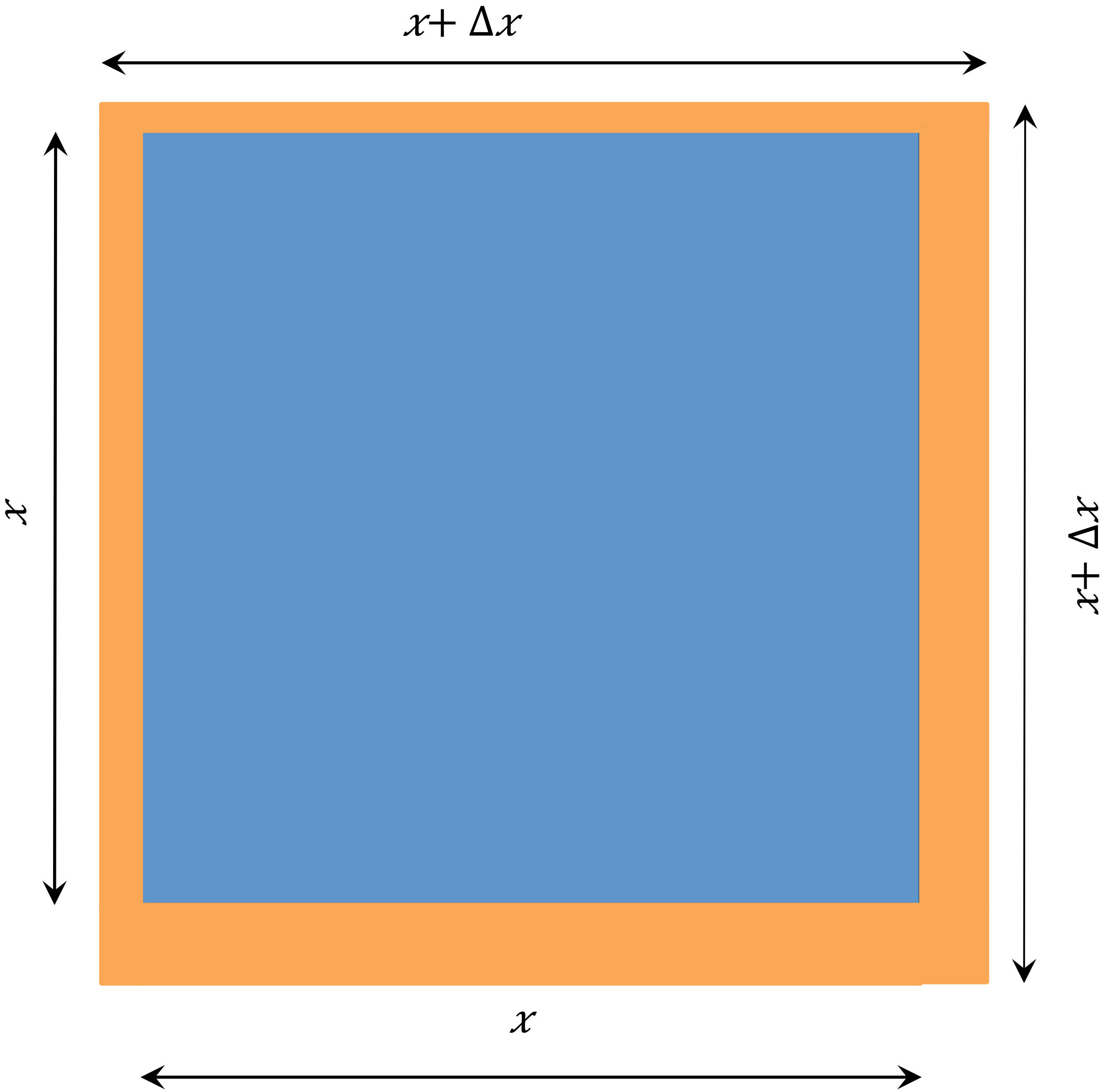
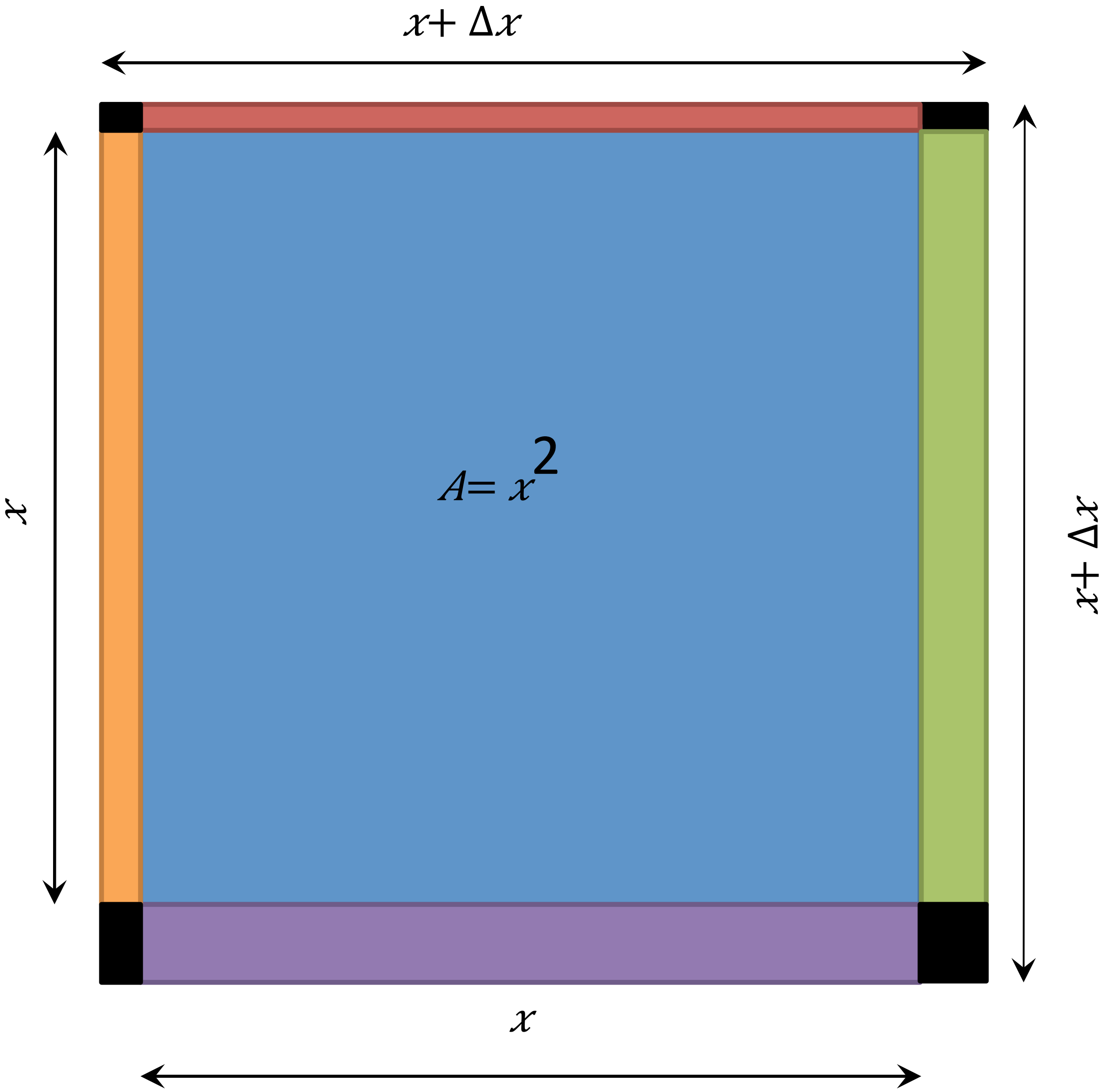
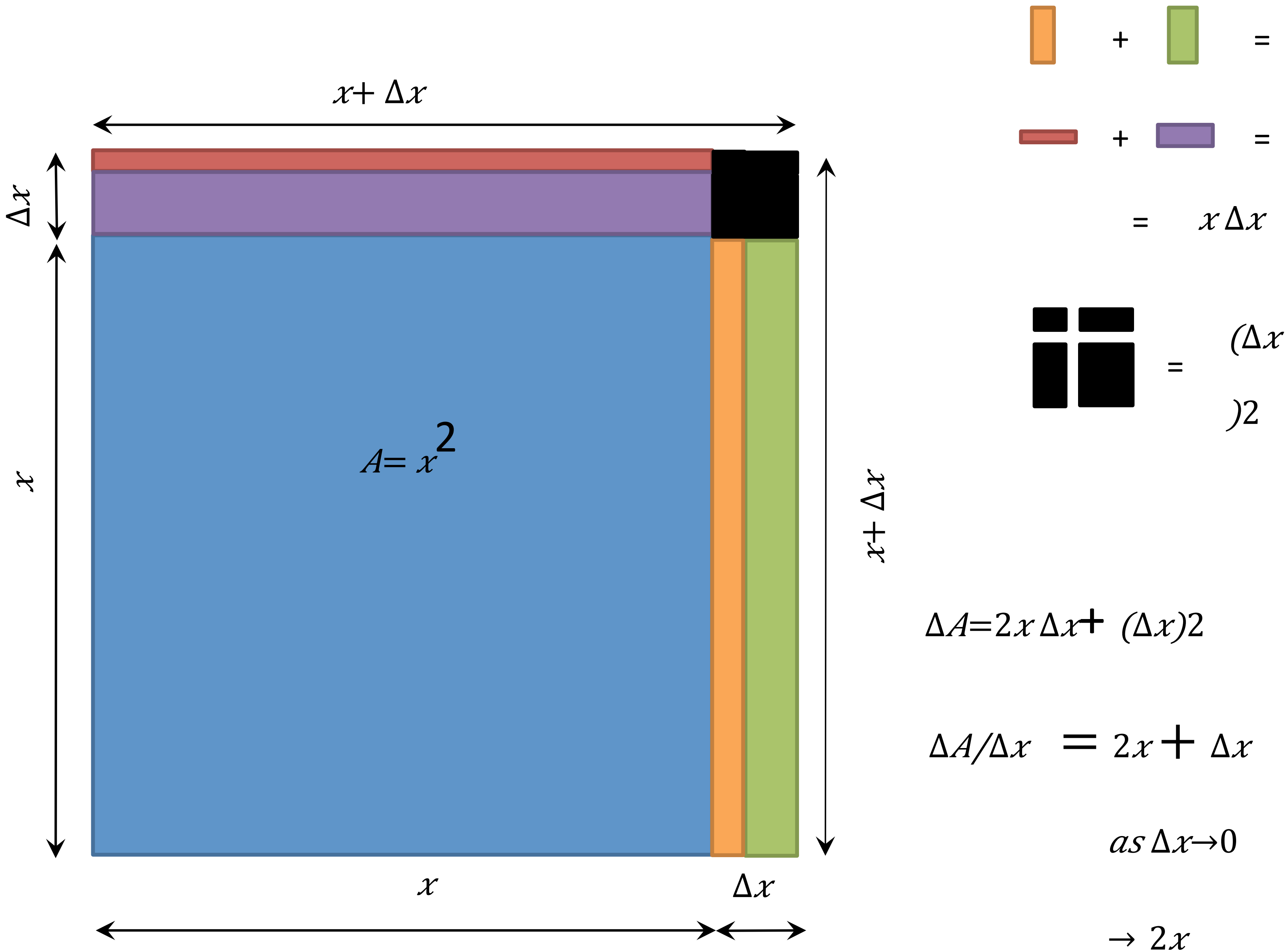
10:28 AM - 5 Oct 13 · Details



$x+\Delta x$
x
$x+\Delta x$
x


$x + \Delta x$
x
$A = x^2$
$x + \Delta x$
x



x+ Δx
Δx
x
A= x 2
x+ Δx
x
Δx
+ =
+ =
= x Δx
= (Δx)2
ΔA=2x Δx+ (Δx)2
ΔA/Δx = 2x + Δx
as Δx→0
→ 2x

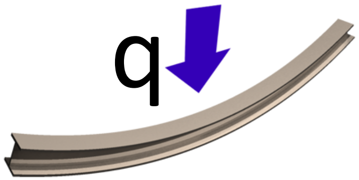
Peter Rowlett @peterrowlett 5 Oct

@johndavidread what about x^4?

Expand

john read @johndavidread 5 Oct

@peterrowlett same theory works for a cube - add 3 square faces. Then extrapolate in your imagination to a hypercube and add four cubes on.



$(EI d^2 w/dx^2) = q$

Leonard Euler 1707 - 1783

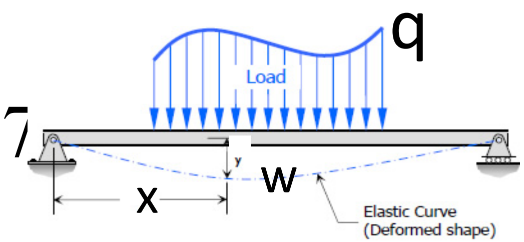
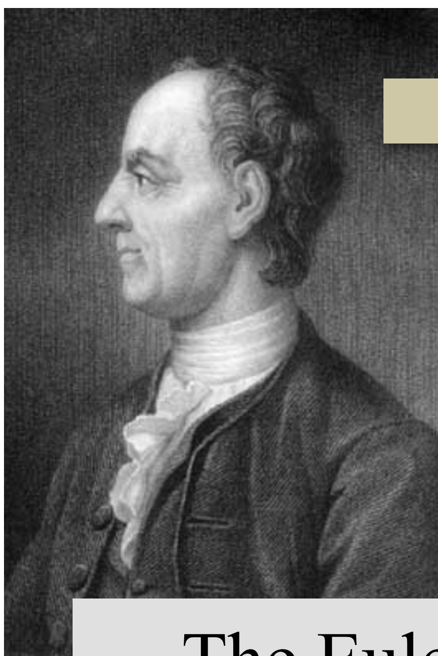
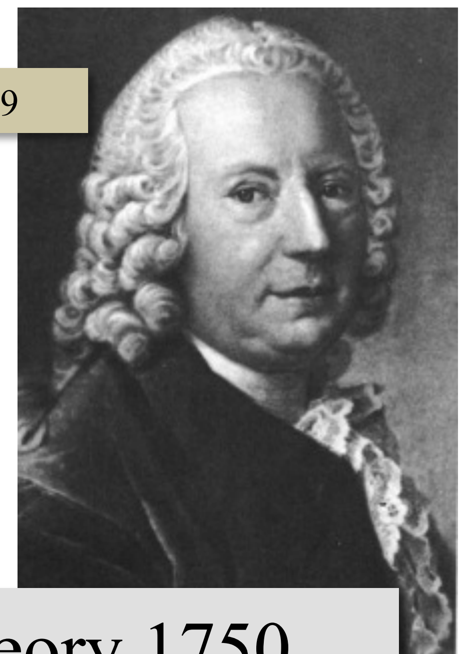


Figure: Elastic curve

Daniel Bernoulli 1700 - 1782



Eiffel Tower under construction 1887-1889



The Euler Bernoulli Beam Theory 1750

$\int q \sim v$

Shear force

$\int v \sim m$

Bending moment

$\int m \sim s$

slope

$\int s \sim w$

deflection

$$EI \frac{d^4 w}{dx^4} = -q(x)$$

$$EI \frac{d^3 w}{dx^3} = V(x) = -\int q(x)\, dx + C_1$$

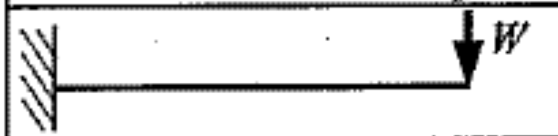
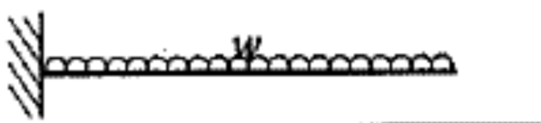
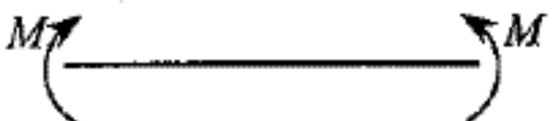
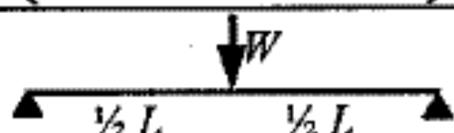
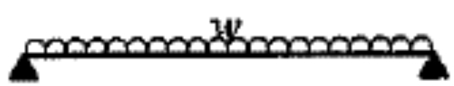
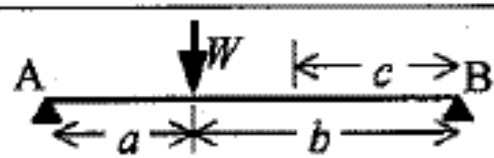
$$EI \frac{d^2 w}{dx^2} = M(x) = -\int dx \int q(x)\, dx + C_1 x + C_2$$

$$EI \frac{dw}{dx} = EI\,\theta(x) = -\int dx \int dx \int q(x)\, dx + \frac{1}{2} C_1 x^2 + C_2 x + C_3$$

$$EI w(x) = -\int dx \int dx \int dx \int q(x)\, dx + \frac{1}{6} C_1 x^3 + \frac{1}{2} C_2 x^2 + C_3 x + C_4$$

The four constants of integration may be determined from the boundary conditions. These conditions include (*a*) the conditions imposed on the deflection or slope of the beam by its supports (cf. Sec. 8.2), and (*b*) the condition that V and M be zero at the free end of a cantilever beam, or that M be zero at both ends of a simply supported beam (cf. Sec. 7.3). This has been illustrated in Fig. 8.30.

BEAM BENDING

L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \le b, \quad c = \sqrt{\frac{1}{3} b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\,\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

Example 8.02

The simply supported prismatic beam AB carries a uniformly distributed load w per unit length (Fig. 8.10). Determine the equation of the elastic curve and the maximum deflection of the beam.

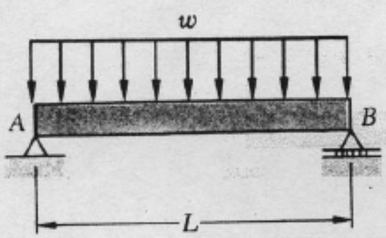


Fig. 8.10

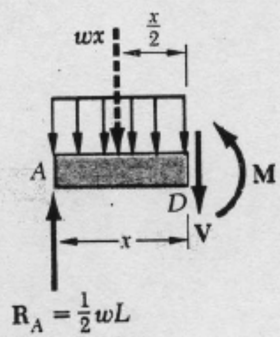


Fig. 8.11

Drawing the free-body diagram of the portion AD of the beam (Fig. 8.11), and taking moments about D, we find that

$$M = \tfrac{1}{2}wLx - \tfrac{1}{2}wx^2 \tag{8.12}$$

Substituting for M into Eq. (8.4) and multiplying both members of this equation by the constant EI, we write

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + \frac{1}{2}wLx \tag{8.13}$$

Integrating twice in x, we have

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{4}wLx^2 + C_1 \tag{8.14}$$

$$EI\,y = -\frac{1}{24}wx^4 + \frac{1}{12}wLx^3 + C_1x + C_2 \tag{8.15}$$

Observing that $y = 0$ at both ends of the beam (Fig. 8.12), we first let $x = 0$ and $y = 0$ in Eq. (8.15) and obtain $C_2 = 0$. We then make $x = L$ and $y = 0$ in the same equation and write

$$0 = -\tfrac{1}{24}wL^4 + \tfrac{1}{12}wL^4 + C_1L$$
$$C_1 = -\tfrac{1}{24}wL^3$$

Carrying the values of C_1 and C_2 back into Eq. (8.15), we obtain the equation of the elastic curve:

$$EI\,y = -\frac{1}{24}wx^4 + \frac{1}{12}wLx^3 - \frac{1}{24}wL^3x$$

or

$$y = \frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x) \tag{8.16}$$

Substituting into Eq. (8.14) the value obtained for C_1, we check that the slope of the beam is zero for $x = L/2$ and that the elastic curve has a minimum at the midpoint C of the beam (Fig. 8.13). Letting $x = L/2$ in Eq. (8.16), we have

$$y_C = \frac{w}{24EI}\left(-\frac{L^4}{16} + 2L\frac{L^3}{8} - L^3\frac{L}{2}\right) = -\frac{5wL^4}{384EI}$$

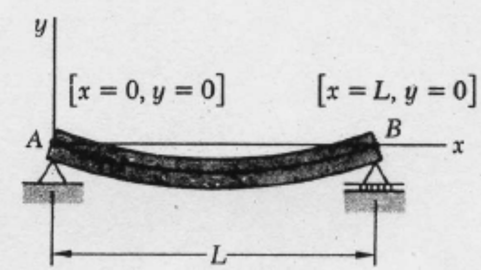


Fig. 8.12

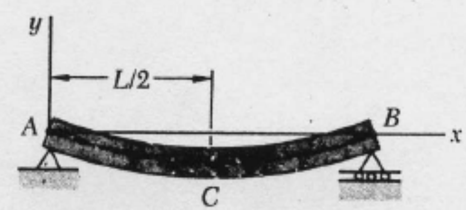


Fig. 8.13

The maximum deflection or, more precisely, the maximum absolute value of the deflection, is thus

$$|y|_{max} = \frac{5wL^4}{384EI}$$

For $q(x)$ = a constant, setting $q = 1$, and setting both $E = 1$ and $I = 1$
and for a unit length beam $l = 1$

$$d^4 w/dx^4 = 1$$

For q(x) = a constant, setting q = 1 , and setting both E = 1 and I = 1
and for a unit length beam l = 1

$$d^4 w/dx^4 = 1$$

$$/24\ x^4 + 1/6\ c_1\ x^3 + 1/2\ c_2\ x^2 + c_3\ \mathrm{x} + c_4$$

For q(x) = a constant, setting q = 1 , and setting both E = 1 and I = 1
and for a unit length beam l = 1

$$d^4 w/dx^4 = 1$$

$$/24\ x^4 + 1/6\ c_1 x^3 + 1/2\ c_2 x^2 + c_3 \mathrm{x} + c_4$$

$$= 1/4!\ x^4 + 1/3!\ c_1 x^3 + 1/2!\ c_2 x^2 + 1/1!\ c_3 x^1 + 1/0!\ c_4$$

0

At the centre of the beam where x = ½

$$w=\frac{1}{4!}\left(\frac{1}{2}\right)^{4}+\frac{1}{3!}c_{1}\left(\frac{1}{2}\right)^{3}+\frac{1}{2!}c_{2}\left(\frac{1}{2}\right)^{2}+\frac{1}{1!}c_{3}\left(\frac{1}{2}\right)^{1}+\frac{1}{0!}c_{4}\left(\frac{1}{2}\right)^{0}$$

At the centre of the beam where x = ½

$$w=\frac{1}{4!}\left(\frac{1}{2}\right)^4+\frac{1}{3!}c_1\left(\frac{1}{2}\right)^3+\frac{1}{2!}c_2\left(\frac{1}{2}\right)^2+\frac{1}{1!}c_3\left(\frac{1}{2}\right)^1+\frac{1}{0!}c_4\left(\frac{1}{2}\right)^0$$

$$w=\frac{1}{2^4\,4!}+\frac{1}{2^3\,3!}c_1+\frac{1}{2^2\,2!}c_2+\frac{1}{2^1\,1!}c_3+\frac{1}{2^0\,0!}c_4$$

At the centre of the beam where x = ½

$$w=\frac{1}{4!}\left(\frac{1}{2}\right)^4+\frac{1}{3!}c_1\left(\frac{1}{2}\right)^3+\frac{1}{2!}c_2\left(\frac{1}{2}\right)^2+\frac{1}{1!}c_3\left(\frac{1}{2}\right)^1+\frac{1}{0!}c_4\left(\frac{1}{2}\right)^0$$

$$w=\frac{1}{2^4\,4!}+\frac{1}{2^3\,3!}c_1+\frac{1}{2^2\,2!}c_2+\frac{1}{2^1\,1!}c_3+\frac{1}{2^0\,0!}c_4$$

$$w=\frac{1}{384}+\frac{1}{48}c_1+\frac{1}{8}c_2+\frac{1}{2}c_3+\frac{1}{1}c_4$$

1,2,8,48,384 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:1,2,8,48,384**

Displaying 1-2 of 2 results found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A000165 Double factorial of even numbers: (2n)!! = 2^n*n!. +20
(Formerly M1878 N0742) 132

1, 2, 8, 48, 384, 3840, 46080, 645120, 10321920, 185794560, 3715891200, 81749606400, 1961990553600, 51011754393600, 1428329123020800, 42849873690624000, 1371195958099968000, 46620662575398912000, 1678343852714360832000, 63777066403145711616000 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,2

COMMENTS a(n) is also the size of automorphism group of the graph (edge graph) of the n dimensional hypercube and also of the geometric automorphism group of the hypercube (the two groups are isomorphic). This group is an extension of an elementary Abelian group (C_2)^n by S_n. (C_2 is the cyclic group with two elements and S_n is the symmetric group) - Avi Peretz (njk(AT)netvision.net.il), Feb 21 2001

Then a(n) appears in the power series: sqrt(1+sin(y))=sum (n>=0,(-1)^floor(n/2)*y^(n)/a(n)) and sqrt((1+cos(y))/2)=sum (n>=0,(-1)^n*y^(2n)/a(2n)) - Benoit Cloitre, Feb 02 2002

The coefficients are the reciprocals of the double factorials !! of n, for n = 0 to 4

Tesseract Graph

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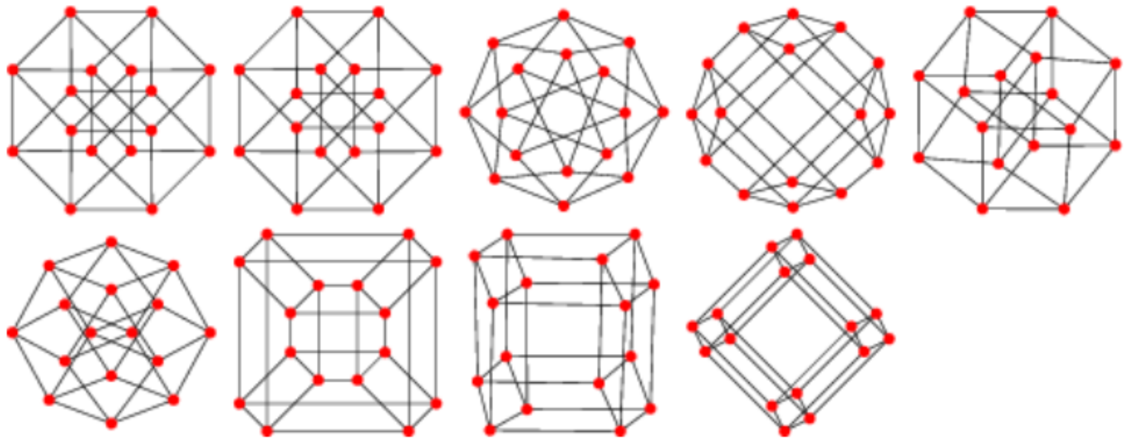
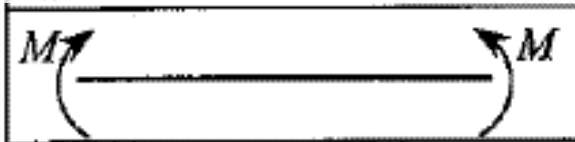
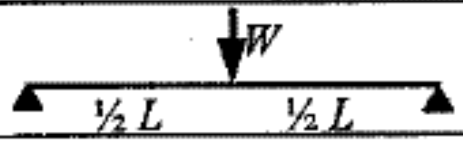
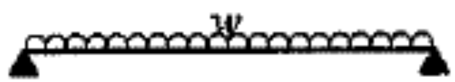


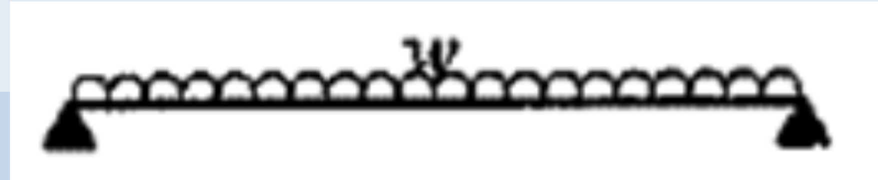
The skeleton of the tesseract, commonly denoted Q_4, is a symmetric quartic graph with girth 4 and diameter 4. The automorphism group of the tesseract is of order $2^7 \cdot 3 = 384$ (Buekenhout and Parker 1998), and it has graph spectrum $(-4)^1\ (-2)^4\ 0^6\ 2^4\ 4^1$, so it has an integral graph. The figures above show several nice embeddings of the tesseract graph, the leftmost of which appears in Coxeter (1973) and a number of which can be found in Carr and Kocay (1999).

It is implemented in *Mathematica* as `GraphData["TesseractGraph"]`.

The tesseract graph is isomorphic to the 4-Hadamard graph.

BEAM BENDING

L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \leq b, \quad c = \sqrt{\tfrac{1}{3}b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

$q = 1 = 0!$	 	
$v = x^{1}/1! + c_{1}\, x^{0}/0!$		
$m = x^{2}/2! + c_{1}\, x^{1}/1! + c_{2}\, x^{0}/0!$	$m_{0} = 0$	$c_{2} = 0$
	$m_{1} = 0$	$c_{1} = -1/2!$
$s = x^{3}/3! + c_{1}\, x^{2}/2! + c_{2}\, x^{1}/1! + c_{3}\, x^{0}/0!$		
$w = x^{4}/4! + c_{1}\, x^{3}/3! + c_{2}\, x^{2}/2! + c_{3}\, x^{1}/1! + c_{4}\, x^{0}/0!$	$w_{0} = 0$	$c_{4} = 0$
	$w_{1} = 0$	$c_{3} = -1/4! + 1/2!\,3! = 1/4!$
$w_{1/2} = 1/4!2^{4} - 1/3!2^{3}\,2! + 0 + 1/1!2^{1}\,4! + 0$	$w_{1/2} = 1/3\,2^{7} - 1/3\,2^{5} + 1/3\,2^{4}$	$w_{1/2} = 1/384 - 1/96 + 1/48$
$w_{1/2} = 2^{0}/4!2^{4} - 2^{2}/4!2^{4} + 2^{3}/4!2^{4}$	$w_{1/2} = 2^{3} - 2^{2} + 2^{0}/4!2^{4} = 5/384$	