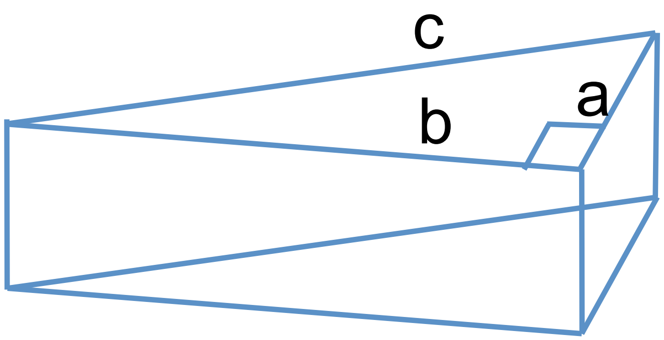
Solving Mathematical Problems by Physical Reasoning[†]

Francis Hunt
fhhunt@glam.ac.uk

[†] drawn from Mark Levi's *The Mathematical Mechanic*

Pythagorean Fish Tank



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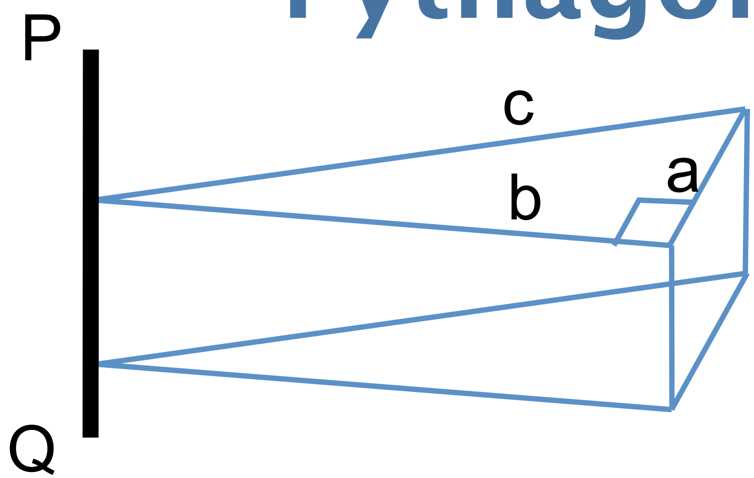

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   (a right-angled triangular prism)

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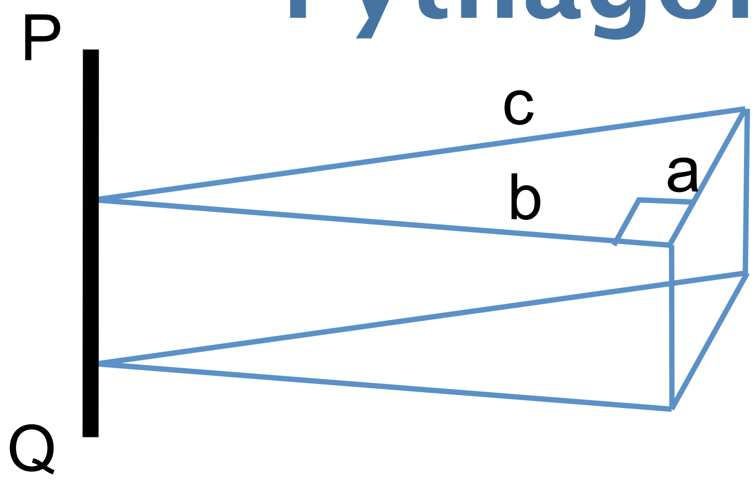

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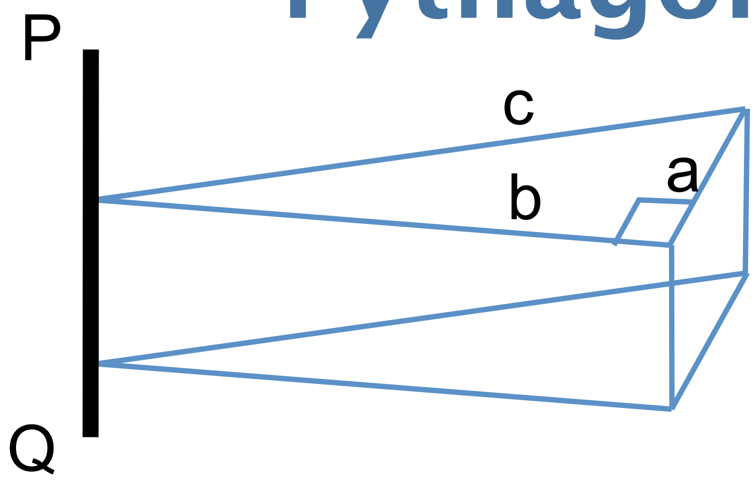

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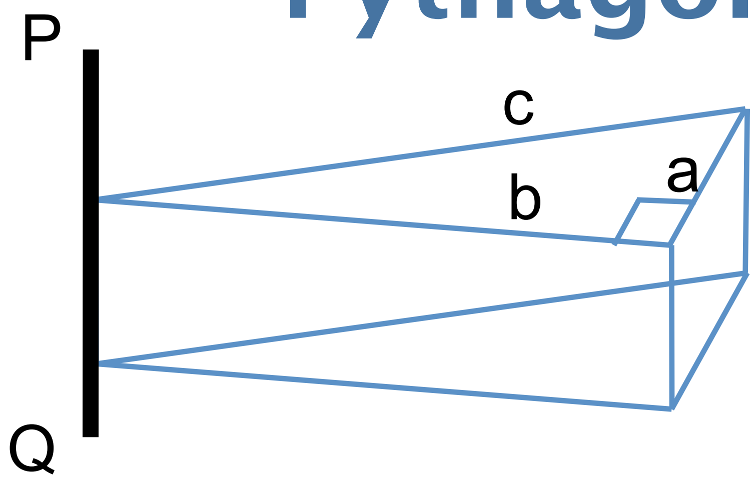

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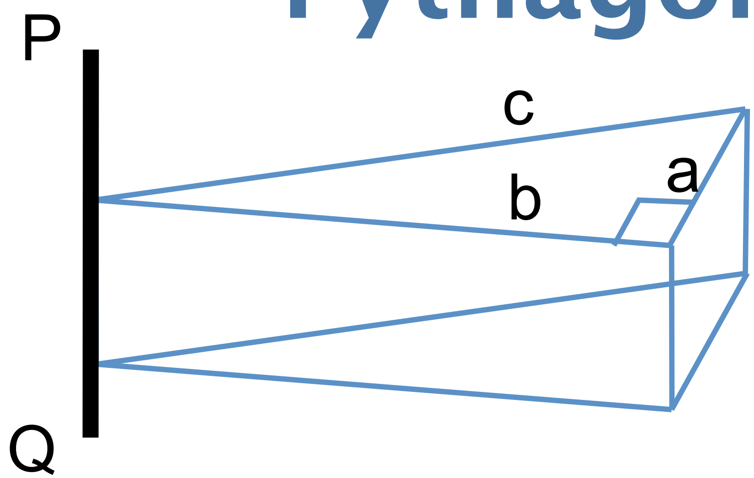
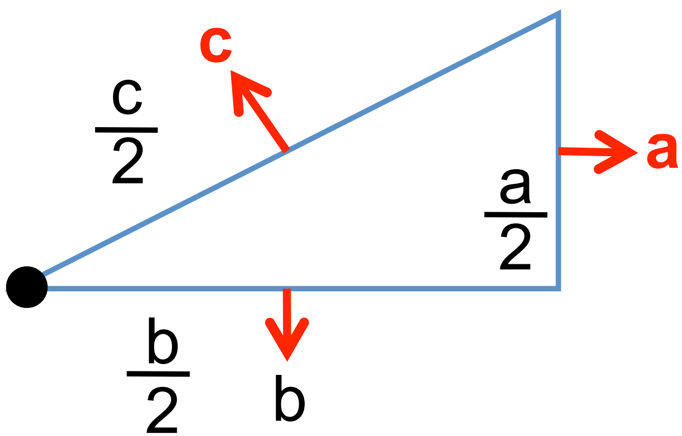


View from above



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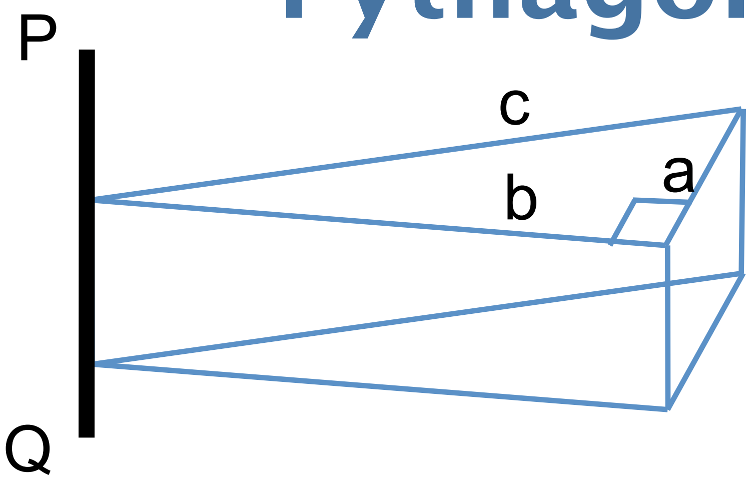

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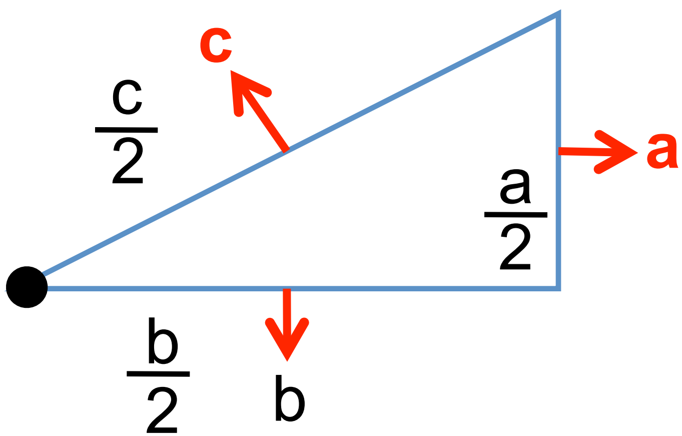

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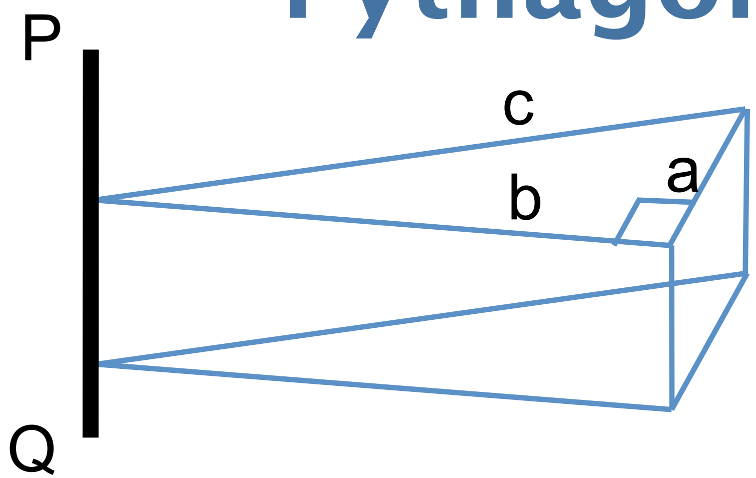

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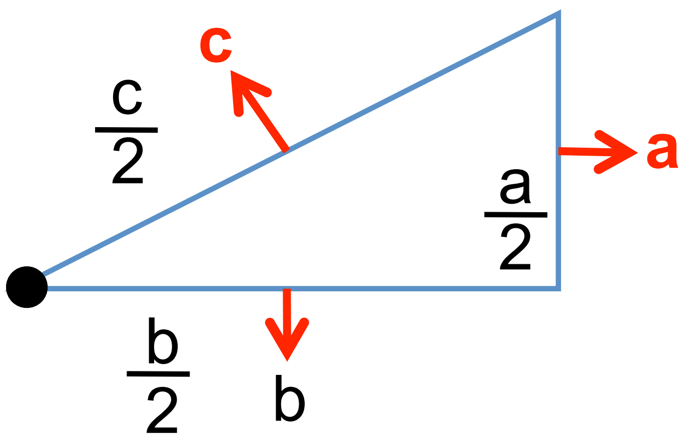

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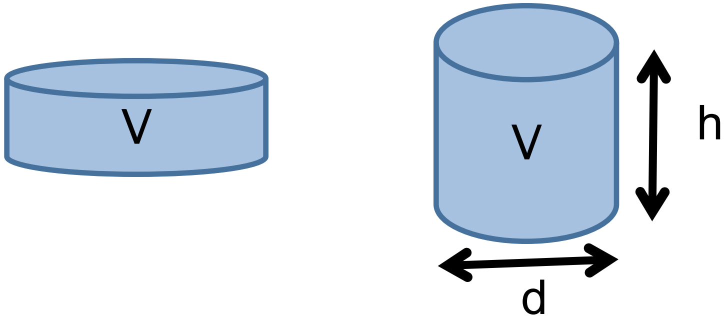
$$a \times a/2 \quad + \quad b \times b/2 = c \times c/$$

Minimal surface area can



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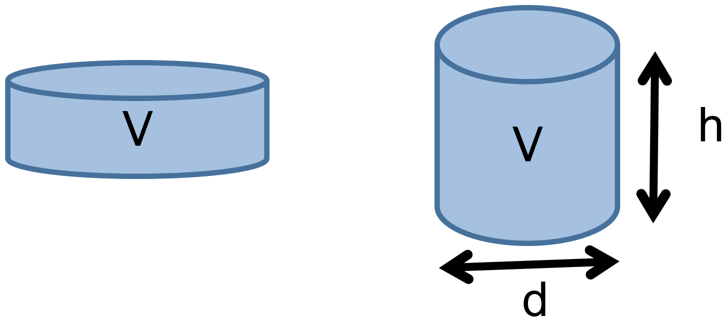
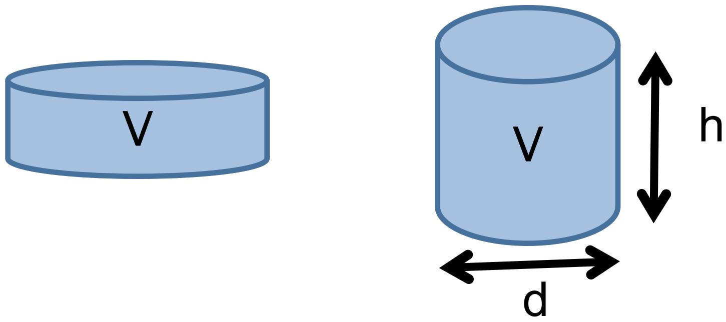


1. Take a frictionless Rubik's can of constant volume V

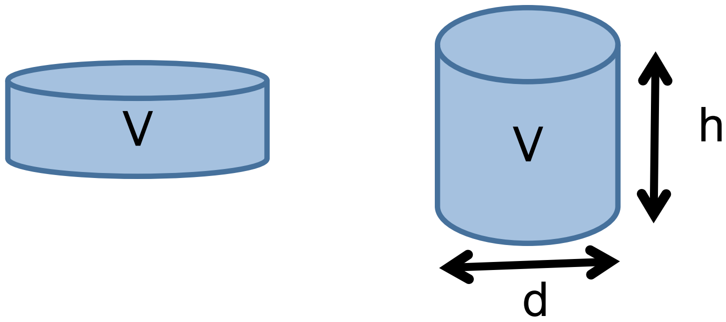
Minimal surface area can



In equilibrium

1. Take a frictionless Rubik's can of constant volume V
2. Coat in a soap film, surface tension σ, energy σA, where A is the surface area

Minimal surface area can



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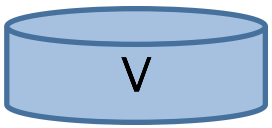

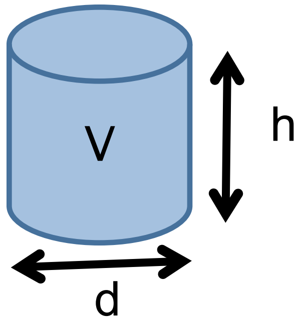

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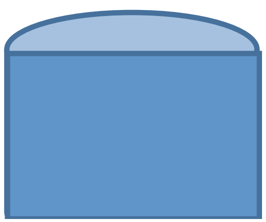

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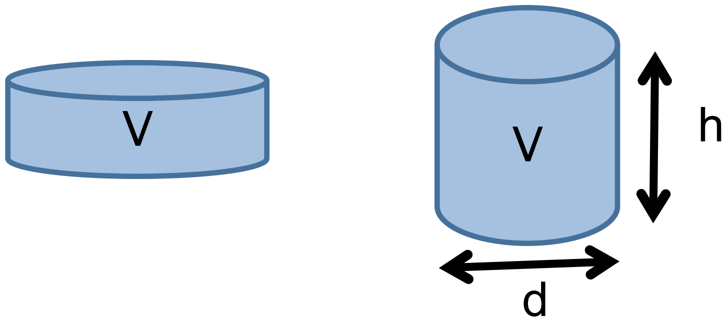

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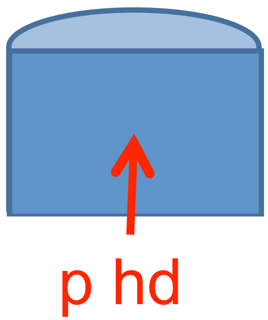

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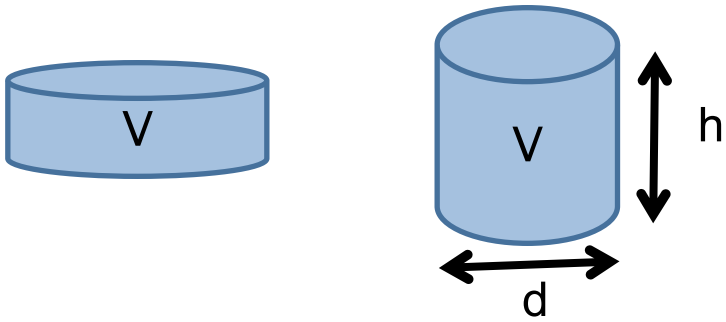

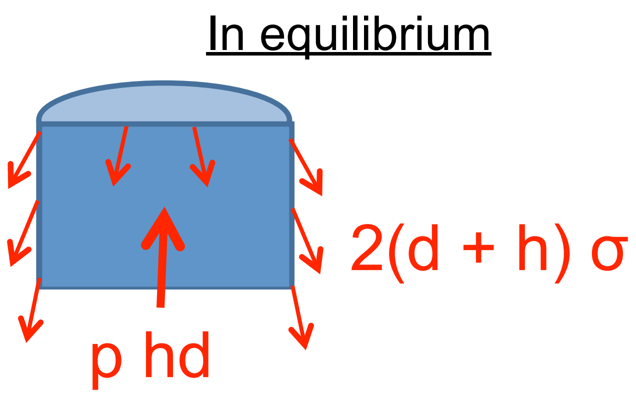

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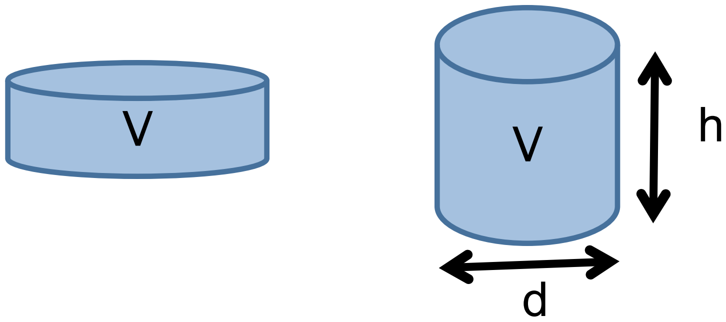

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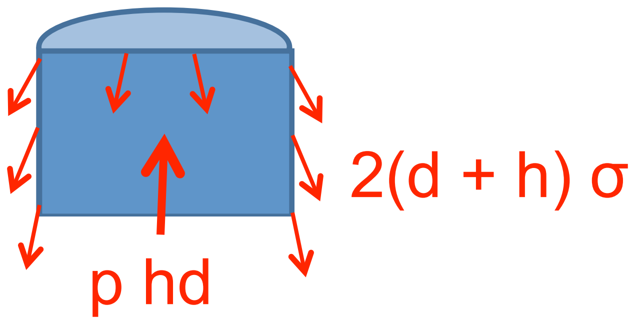

1. Take a frictionless Rubik's can of constant volume V
2. Coat in a soap film, surface tension σ, energy σA, where A is the surface area
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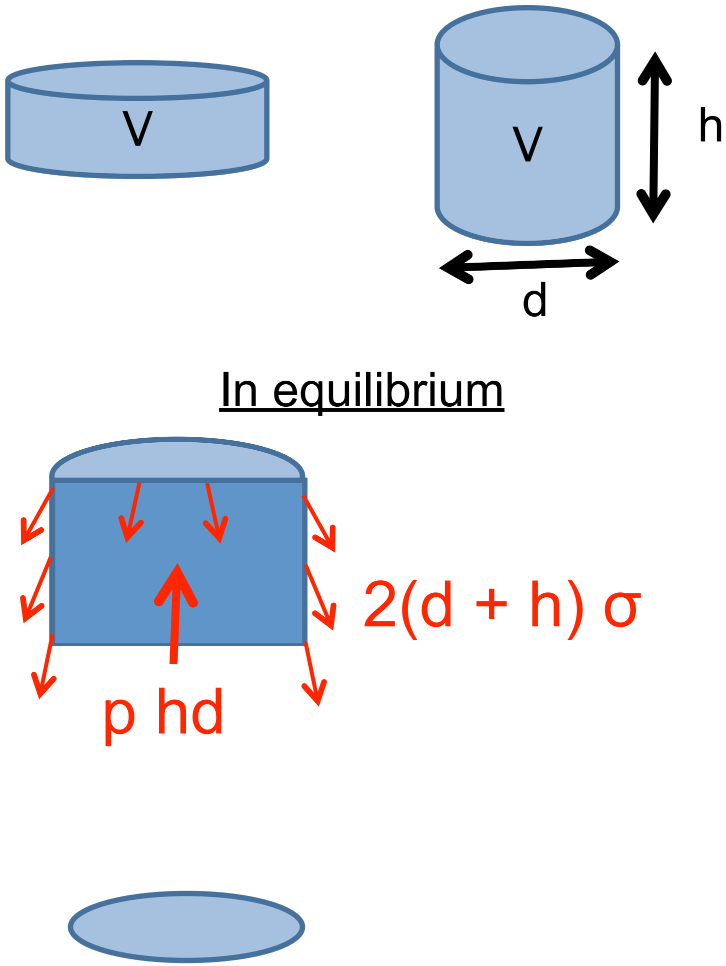
4. Examine the balance of forces

Minimal surface area can



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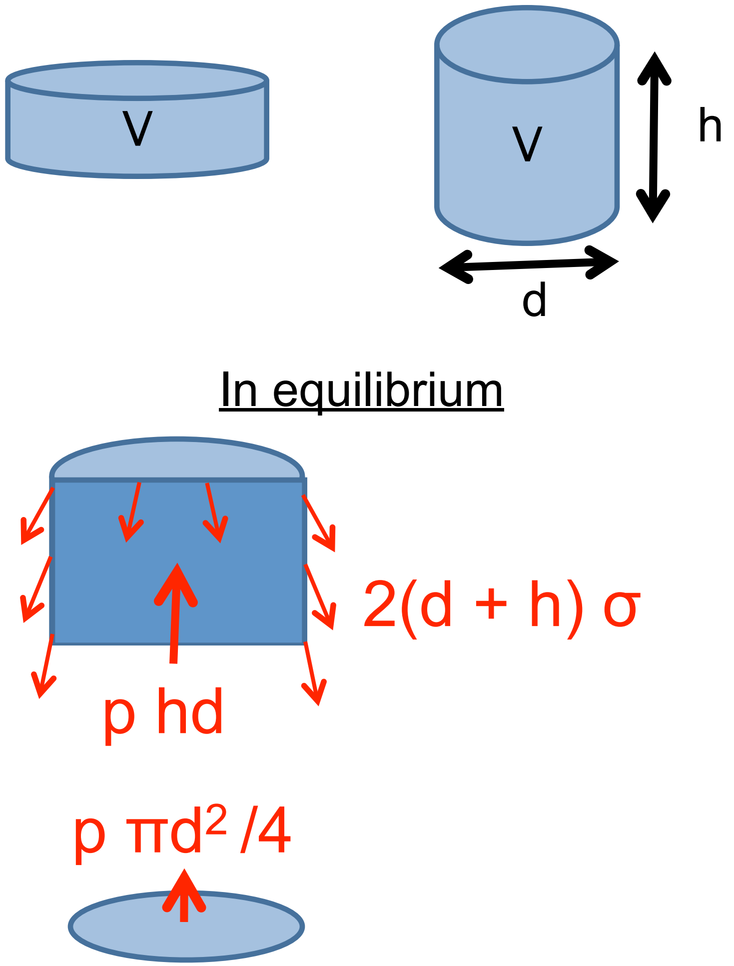

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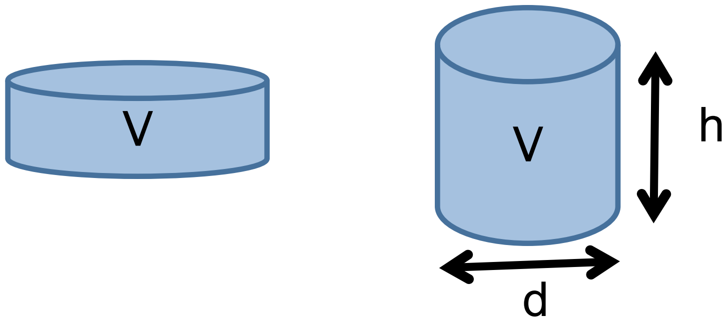

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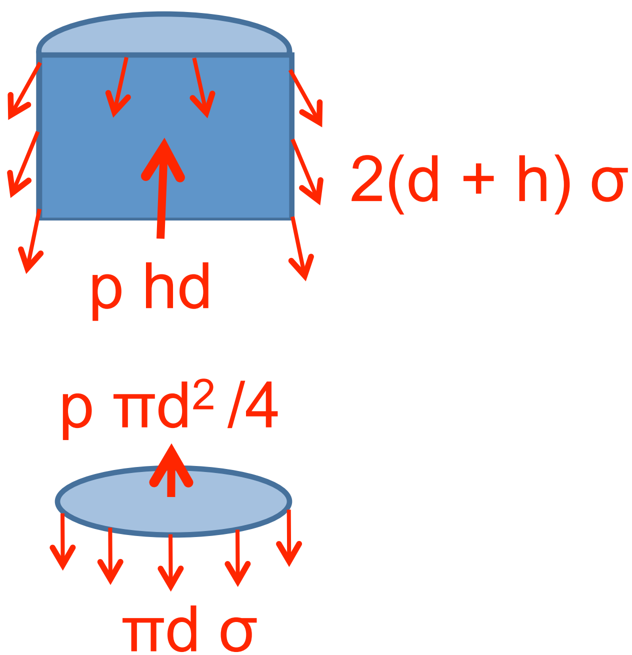

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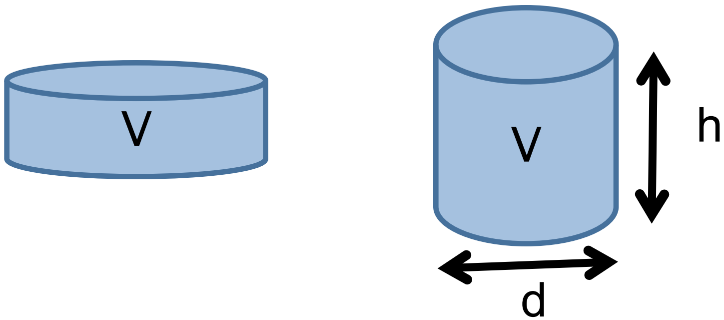

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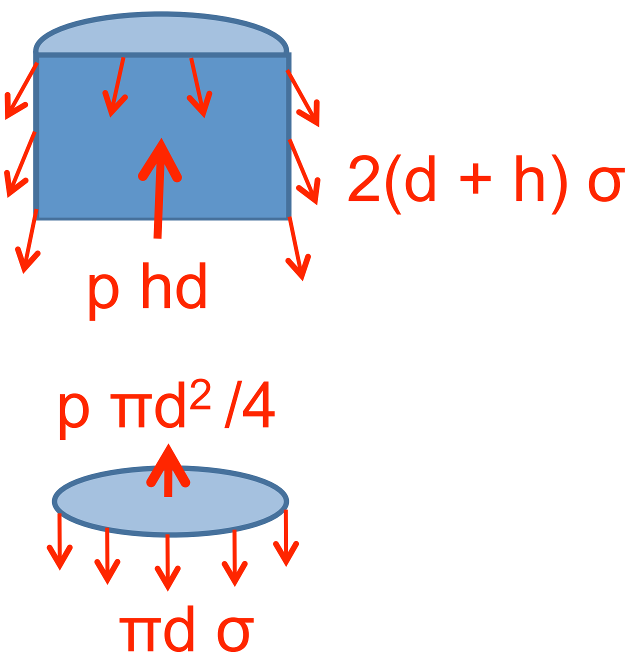

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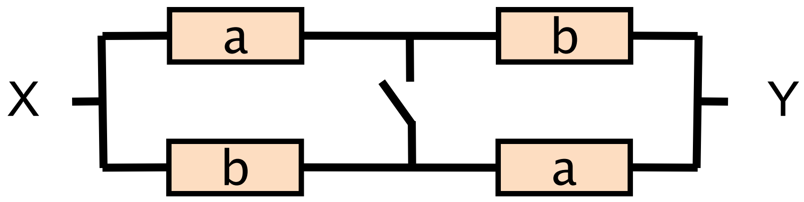
$$phd = 2(d+h)\,\sigma$$

Arithmetic mean ≥ Geometric mean



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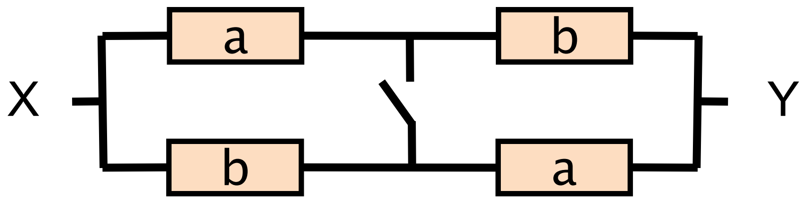
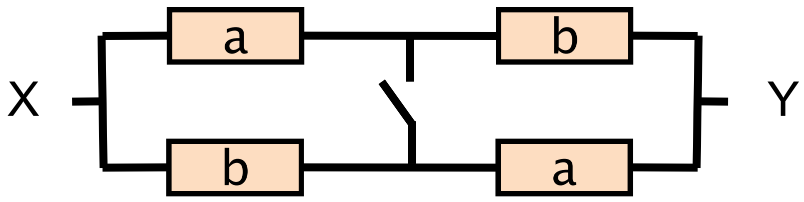
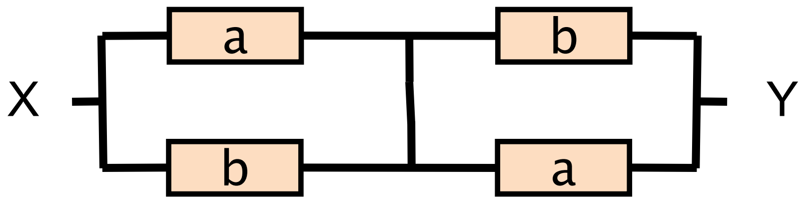


1. Resistance between X and Y with switch open is

$$(a + b)/2$$

Arithmetic mean ≥ Geometric mean





1. Resistance between X and Y with switch open is

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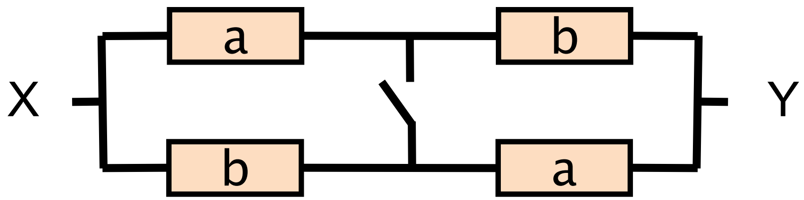
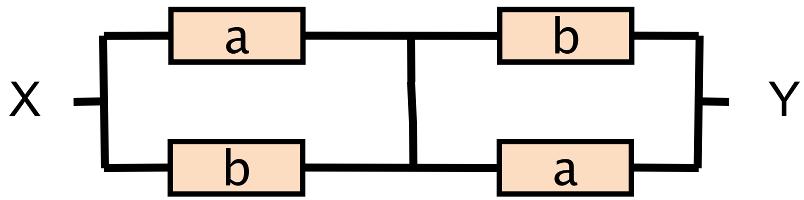
2. Resistance between X and Y with the switch closed is

   $$1/(1/a + 1/b) \times 2$$

Arithmetic mean ≥ Geometric mean





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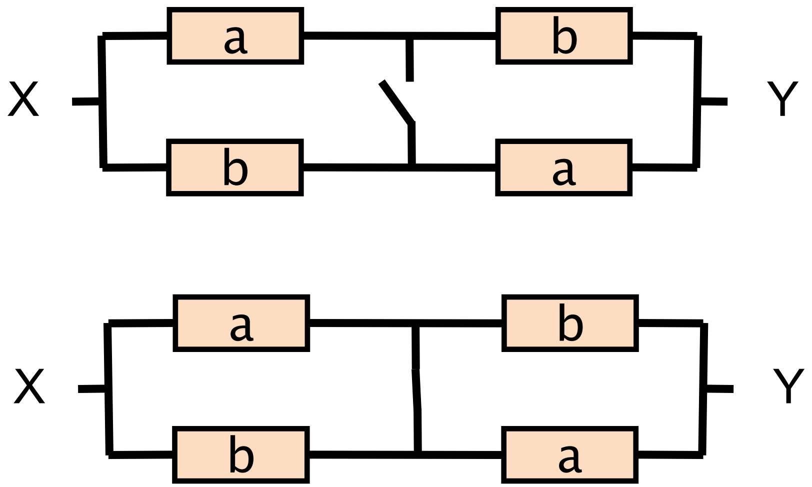
2. Resistance between X and Y with the switch closed is

   $$1/(1/a + 1/b) \times 2$$
   $$= 2ab/(a+b)$$

Arithmetic mean ≥ Geometric mean



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   $(a + b)/2$

2. Resistance between X and Y with the switch closed is

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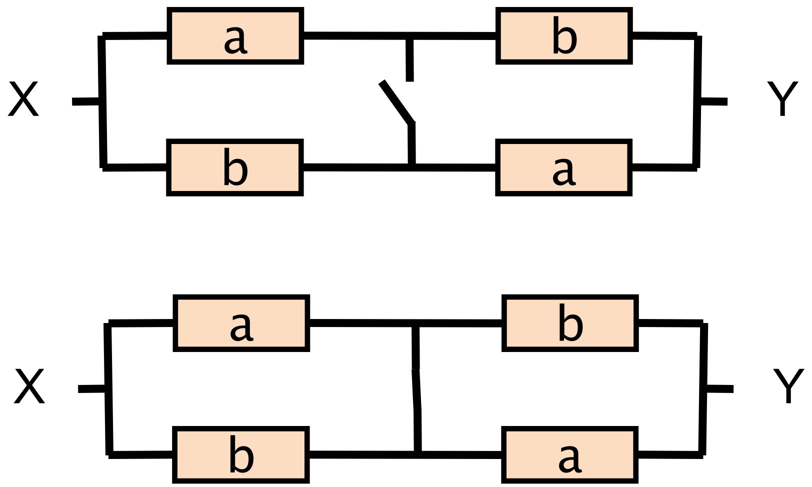
   $= 2ab/(a+b)$

3. Closing switch didn't increase resistance so

Arithmetic mean ≥ Geometric mean



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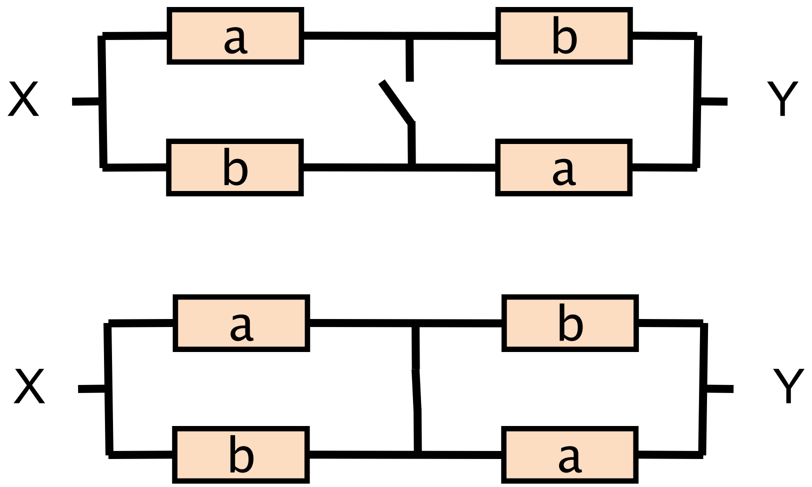
3. Closing switch didn't increase resistance so

$$(a+b)/2 \geq 2ab/(a+b)$$

Arithmetic mean ≥ Geometric mean



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   or